Market Making Contracts, Firm Value, and the IPO Decision*

Hendrik Bessembinder
University of Utah

Jia Hao
University of Michigan

Kuncheng Zheng
University of Michigan

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Abstract:
We examine the effects of secondary market liquidity on firm value and the decision to conduct an Initial Public Offering (IPO). Competitive liquidity provision can lead to market failure as the IPO either does not occur or the IPO price is discounted to reflect that some welfare-enhancing secondary trades do not occur. Market failure arises when uncertainty regarding fundamental value and asymmetric information are large in combination. In these cases, firm value and social welfare are improved by a contract where the firm engages a Designated Market Maker (DMM) to enhance liquidity. Our model implies that such contracts represent a market solution to a market imperfection, particularly for small growth firms. In contrast, proposals to encourage IPOs by use of a larger tick size are likely to be counterproductive.

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1. Introduction

Modern stock markets rely on competing limit orders to supply liquidity. In contrast to the designated “specialists” who in past decades coordinated trading on the flagship New York Stock Exchange (NYSE), limit order traders are typically not obligated to supply liquidity or otherwise facilitate trading. The desirability of such endogenous liquidity provision has recently been questioned. Mary L. Shapiro, former chair of the United States Securities and Exchange Commission (SEC) stated “The issue is whether the firms that effectively act as market makers during normal times should have any obligation to support the market in reasonable ways in tough times.”\(^1\) A joint SEC-CFTC advisory committee asserted that “incentives to display liquidity may be deficient in normal market, and are seriously deficient in turbulent markets.”\(^2\) Related, the SEC Advisory Committee on Small and Emerging Companies recently recommended an increase the minimum price increment or “tick size” for smaller exchange-listed companies to “increase their liquidity and facilitate IPOs and capital formation.”\(^3\) Each of these policy initiatives is predicated on the notion that competitive market forces do not always supply sufficient liquidity.

In this paper, we introduce a simple model of secondary market illiquidity and its effect on stock prices and incentives to conduct IPOs. Our model shows that competitive secondary market liquidity provision can indeed lead to market failure. Failure occurs in particular for those firms or at those times when the combination of uncertainty regarding asset value and the likelihood of information asymmetry is high.

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As a potential cure, we focus on contracts by which the firm hires a “Designated Market Maker” (DMM) to enhance liquidity. Such contracts are observed on several stock markets, including the leading markets in Germany, France, Italy, the Netherlands, Sweden, and Norway. The most frequently observed obligation is a “maximum spread” rule, which requires the DMM to keep the bid-ask spread (the difference between the lowest price for an unexecuted sell order and the highest price for an unexecuted buy order) within a specified width, in exchange for a periodic payment from the firm. DMM contracts of this type are currently prohibited in the U.S. by FINRA rule 5250, which “prohibits any payments by an issuer or an issuer’s affiliates and promoters … for publishing a quotation, acting as a market maker or submitting an application in connection therewith.” The NYSE and Nasdaq markets have both recently requested partial exemptions from rule 5250, to allow DMM contracts for certain Exchange Traded Funds. Some commentators have criticized these proposals on the grounds that DMM contracts distort market forces.

Our model shows that insufficient liquidity in competitive secondary markets can lead to complete market failure, where the firm chooses to not conduct the IPO even though social welfare would be enhanced by doing so, or partial market failure, where the IPO is completed at a price that is discounted to reflect that secondary market illiquidity will dissuade some efficient trading. We show that a DMM contract by which the firm pays a market maker a fixed fee in exchange for narrowing the bid-ask spread can cure these market failures. Such a contract reduces market maker trading profits and/or imposes expected trading losses, but also increases the equilibrium IPO price, as investors take

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5 Such obligations typically bind. For example, Anand, Tanggaard, and Weaver (2009) study DMM agreements on the Stockholm Stock Exchange and document that the contracted maximum spreads are typically narrower than the average spread that prevailed prior to the introduction of DMMs. In contrast, the “Supplemental Liquidity Suppliers” and “Designated Market Makers” currently employed on the NYSE are only obligated to enter orders that match the best existing prices a certain percentage of the time, and are not required to improve on the best prices from public limit orders.
6 FINRA Regulatory Notice 09-60.
into account the benefits of being able to subsequently trade at lower cost. Notably, the increase in the
IPO price can exceed the requisite payment to compensate the DMM, thereby increasing the firm’s net
proceeds. The DMM contract comprises a potential market solution to a potential market failure.

In our model, as in the classic analysis of Glosten and Milgrom (1985), illiquidity is attributable
to information asymmetry. A key point of perspective is that, while informational losses comprise a
private cost to liquidity suppliers, these are zero-sum transfers rather than a cost when aggregated across
all agents. Competitive bid-ask spreads compensate liquidity suppliers for their private losses to better
informed traders, and are therefore wider than the net social cost of completing trades. A maximum
spread rule can improve social welfare and firm value because more investors will choose to trade when
the spread is narrower. This increased trading enhances allocative efficiency and firm value.

The model generates cross-sectional predictions. It implies that the efficacy of DMM contracts
depends on the interaction of uncertainty and asymmetric information regarding the fundamental value
of the firm’s assets. In the absence of asymmetric information, competitive liquidity provision is optimal,
regardless of the degree of uncertainty regarding underlying value. In contrast, the combination of a
high probability of information asymmetry and high uncertainty regarding fundamental value leads to
potential market failure, and to improved firm value and social welfare from a DMM agreement. This
combination is likely to arise for smaller, younger, and growth-oriented firms in particular. Further, the
model implies that reductions in liquidity that are attributable to real or perceived increases in
information asymmetry are economically inefficient, providing economic justification for a contractual
requirement to enhance liquidity at times of high perceived information asymmetries.

The model has regulatory implications. If the market for liquidity provision is competitive, then
contracts that required further narrowing of bid-ask spreads will impose expected trading losses on and
require side payments to the DMM. A regulatory requirement that certain liquidity suppliers provide
liquidity beyond competitive levels without compensation could lead to exit from the industry and
ultimately be counterproductive to the goal of enhancing liquidity.
Our analysis is also relevant to regulatory initiatives related to the minimum price increment. The U.S. Congress recently directed the SEC to reassess the effects the 2001 decimalization of the U.S. equity markets, which reduced the minimum price increment to one cent. The SEC’s advisory committee has recommended larger tick sizes for smaller exchange-listed companies, to increase market making profits, attract additional liquidity supply, and encourage IPOs. Our model indicates that this approach is also likely to be counterproductive. Monopoly profits in liquidity provision reduce IPO prices as investors take into account increased secondary market trading costs. Lower IPO prices, in turn, can lead to market failure as firms elect to forgo the IPO entirely. In contrast, our model implies that the DMM contracts considered here can lead to higher IPO prices and facilitate IPOs.

II. The Related Literature

The literature on market making is vast. However, in most models the emphasis is on endogenous liquidity provision, i.e. on dealer and trader behavior in the absence of any specific obligation to supply liquidity. Among the few exceptions, Venkataraman and Waisburd (2007) consider the effect of a DMM in a periodic auction market characterized by a finite number of investors. The DMM in their model is an additional trader present in every round of trading, thereby improving risk sharing. Sabourin (2006) presents a model where a non-compensated market maker is introduced to an imperfectly competitive limit order market. In her model, the continuous presence of a market maker causes some limit order traders to substitute to market orders, allowing the possibility of wider spreads with a market maker, for some parameters.

Among the empirical studies of DMMs, Venkataraman and Waisburd (2007), Anand, Tanggaard, and Weaver (2009), Skjeltorp and Odegaard (2014), and Menkveld and Wang (2013) study the introduction of DMMs on Euronext-Paris, the Stockholm Stock Exchange, the Oslo Stock Exchange, and Euronext-Amsterdam, respectively. In contrast to Sabourin (2006), each study reports

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9 The directive is contained in Section 106(b) of the 2012 Jumpstart Our Business Startups Act.
improvements in liquidity associated with DMM introduction, and each documents positive stock valuation effects on announcement of DMM introduction. Anand and Venkataraman (2013) and Petrella and Nimalendran (2003) study markets (the Toronto and Milan Stock Exchanges, respectively) where DMMs operate in parallel with endogenous liquidity providers, each documenting improved market quality associated with the hybrid structure.

While the empirical evidence supports the reasoning that DMMs enhance liquidity and firm value for at least some stocks, the evidence does not clarify the source of the value gain. Amihud and Mendelson’s (1986) model implies that improved liquidity will reduce firms’ cost of capital. However, providing enhanced liquidity is costly, and the DMMs must be compensated for these costs. Our analysis clarifies the economic mechanism by which firm value can be enhanced by more than the cost of compensating the DMM, and provides cross-sectional implications regarding the firms for which such an agreement will be value-enhancing.

III. The Model

We consider a simple three-date model. At $t=0$ the firm considers selling its existing asset to an investor in an IPO. Secondary market trading can occur at $t=1$. The asset is liquidated at $t=2$, paying $R = (1 + \delta)\mu$ or $R = (1 - \delta)\mu$ with equal probability, where $0 < \delta < 1$. All agents are risk neutral and interest rates are zero, so $\mu > 0$ can be viewed as the fundamental value of the asset at $t = 0$. At $t=0$, the firm can elect to (i) complete the IPO while also entering a DMM contract by which the firm makes a fixed payment to market makers in exchange for a commitment to enhance secondary market liquidity, (ii) complete the IPO without entering a DMM contract, or (iii) forgo the IPO.

To motivate the potential IPO, we assume that the firm needs cash. The $t=0$ value of the asset to the firm is $V_e = (1 - \delta)\mu$, where $0 < \delta < 1$. This discount can reflect that the firm’s owners wish to diversify their holdings or need capital to invest in additional projects. If the investor acquires the asset in the IPO she will with probability $\lambda > 0$ be subject to a liquidity shock that reduces her subjective valuation of the asset by $\rho \mu$, where $0 < \rho < 1$, and will with probability $p$ receive private information.
as to whether the $t=2$ liquidation value of the asset will be $H$ or $L$.\textsuperscript{11} The potential liquidity shock and information arrival both occur just prior to $t=1$ secondary market trading. Each random outcome is assumed to be statistically independent of the others.

We assume the presence of $N \geq 2$ risk-neutral market makers who are not subject to liquidity shocks and who compete \textit{a la} Bertrand.\textsuperscript{12} The market makers set the $t=1$ bid quote, $B$, in the knowledge that the investor can be motivated to trade by either liquidity needs or private information. Let $M(B)$ denote the investor’s expected monetary gain (or equivalently the market makers’ expected loss) from secondary market trading, conditional on the bid price, $B$. $M = 0$ with competitive market making, but could be positive if the firm contracts with a market maker to increase the bid price or negative if there were economic rents in market making. Let $q(B)$ denote the probability that the investor will be dissuaded by a low bid price from selling her asset, conditional on a $t=1$ liquidity shock. The expected cost to the investor attributable to secondary market illiquidity is the product of the probability of a liquidity shock, $\lambda$, the probability that the investor does not sell conditional on the shock, $q(B)$, and the magnitude of the shock, $\rho \mu$. Hence, the value of the asset to the investor at $t=0$ is the expected cash flow $\mu$ minus the expected illiquidity cost, plus the expected monetary gain from trading:

$$V_t(B) = \mu - q(B)\lambda \rho \mu + M(B).$$\textsuperscript{13}
Aggregating across the firm, the investor, and market makers, and noting that monetary payments are zero-sum, the enhancement to social welfare from conducting the IPO can be stated as:

\[ W(B) = (V_f(B) - V_e) - M(B) = \mu(\delta - q(B)\lambda\rho). \]  

(W2)

Welfare is enhanced by the IPO if the firm’s need for cash, \( \delta \), is large relative to the costs attributable to secondary market illiquidity, \( q(B)\lambda\rho \). The firm can potentially increase \( V_f \) by contracting with a market maker to increase the bid price, thereby decreasing \( q(B) \). We initially consider model outcomes while assuming that the firm has complete bargaining power, and is able to extract all investor surplus in the form of an IPO price equal to \( V_f \). In this case, firm value maximization and social welfare maximization coincide. In Section V we relax the assumption of complete bargaining power to consider model outcomes when gains are shared through an IPO price that is less than \( V_f \). Table 1 contains a list of the notation used in the model. All proofs not stated in the text are contained in the Appendix.

Table 1: Definition of Notation

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Algebraic Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \mu )</td>
<td>The unconditional expected value of the asset.</td>
<td>( \mu &gt; 0 )</td>
</tr>
<tr>
<td>( \epsilon )</td>
<td>The uncertainty regarding asset value.</td>
<td>( 0 &lt; \epsilon &lt; 1 )</td>
</tr>
<tr>
<td>( H )</td>
<td>The high liquidation value.</td>
<td>( H = (1 + \epsilon)\mu )</td>
</tr>
<tr>
<td>( L )</td>
<td>The low liquidation value.</td>
<td>( L = (1 - \epsilon)\mu )</td>
</tr>
<tr>
<td>( \delta )</td>
<td>The firm’s discount factor, reflecting its need for cash.</td>
<td>( 0 &lt; \delta &lt; 1 )</td>
</tr>
<tr>
<td>( V_f )</td>
<td>The ( t=0 ) value of the asset to the firm.</td>
<td>( V_f = (1 - \delta)\mu )</td>
</tr>
<tr>
<td>( \lambda )</td>
<td>The probability that the investor incurs a liquidity shock.</td>
<td>( \lambda &gt; 0 )</td>
</tr>
<tr>
<td>( \rho )</td>
<td>The fraction by which the investor’s subjective valuation of the asset is reduced when she incurs a liquidity shock.</td>
<td>( 0 &lt; \rho &lt; 1 )</td>
</tr>
<tr>
<td>( p )</td>
<td>The probability that the investor will become privately informed as to whether the liquidation value of the asset will be H or L.</td>
<td></td>
</tr>
<tr>
<td>( B )</td>
<td>The ( t=1 ) bid quote. ( B^* ) denotes the level that maximizes welfare.</td>
<td>( B^* = (1 + \epsilon - \rho)\mu )</td>
</tr>
<tr>
<td>( M(B) )</td>
<td>The investor’s expected monetary gain, or equivalently, the market maker’s expected monetary loss, from ( t=1 ) secondary market trading.</td>
<td></td>
</tr>
<tr>
<td>( C )</td>
<td>The payment from the firm to the market maker.</td>
<td></td>
</tr>
<tr>
<td>( q(B) )</td>
<td>The probability that the investor will choose not to sell her asset, conditional on a ( t=1 ) liquidity shock.</td>
<td></td>
</tr>
<tr>
<td>( V_i )</td>
<td>The ( t=0 ) value of the asset to the investor.</td>
<td>( V_i = \mu - q\lambda\rho\mu + M )</td>
</tr>
<tr>
<td>( \beta )</td>
<td>The firm’s bargaining power in the IPO market.</td>
<td></td>
</tr>
<tr>
<td>( W )</td>
<td>The improvement in aggregate social welfare.</td>
<td>( W = \mu(\delta - q\lambda\rho) )</td>
</tr>
<tr>
<td>( \pi )</td>
<td>The firm’s net gain from conducting the IPO.</td>
<td>( \pi = \beta(V_f - V_e) - C )</td>
</tr>
</tbody>
</table>
\[ \epsilon < \epsilon^* \] LOW RELATIVE FUNDAMENTAL UNCERTAINTY.

\[ \epsilon^* \equiv \frac{2\lambda + (1 - \lambda)p}{2(p + \lambda - p\lambda)} \rho \]

\[ \epsilon^* < \epsilon < \epsilon^* \] INTERMEDIATE RELATIVE FUNDAMENTAL UNCERTAINTY.

\[ \epsilon^{**} \equiv \frac{p + 2(1 - p)\lambda}{p} \rho \]

\[ \epsilon > \epsilon^{**} \] HIGH RELATIVE FUNDAMENTAL UNCERTAINTY.

IV. Model Outcomes with a Competitive IPO market and Competitive Market Making

We first assess the model’s implications when the IPO market and the market for secondary trading are both competitive. In particular, the firm is able to extract all of the investor’s gains from trade in the form of an IPO price equal to the investor’s valuation, \( V_t \), and expected profits to market making in the absence of a DMM agreement are zero.


Consider as a benchmark the case where the probability that the trader becomes privately informed regarding asset value, \( p \), equals zero, implying that market makers suffer no losses due to information asymmetry. In the absence of any other market making costs, the competitive bid quote in the secondary market is \( B = \mu \). The investor’s subjective \( t=1 \) valuation if she suffers a liquidity shock is \( \mu(1 - \rho) \). Since this is less than the bid quote the investor will always sell following a liquidity shock. Setting \( q = 0 \), we have \( V_t = \mu \) and \( W = \delta\mu \), which is the maximum welfare gain attainable in this model. That is, in the absence of illiquidity due to asymmetric information the investor pays full value for the asset in the IPO, the firm completes the IPO for any positive \( \delta \), and the welfare gain is maximized.

IV.B. Outcomes when the Secondary Market is Illiquid.

We now consider a competitive secondary market in the presence of information asymmetry. The zero-expected-profit bid quote is the expected asset value conditional on a sale by the investor, which occurs if the investor’s subjective \( t=1 \) valuation of the asset is less than the market bid quote, \( B \). For some parameter ranges there are two bid quotes consistent with the zero-expected-profit condition, in which case we assume that competition leads to selection of the greater of the two in equilibrium.
Outcomes in this model depend crucially on the interplay of the likelihood of information asymmetry, \( p \), and the degree of uncertainty regarding the liquidation value of the asset, \( \epsilon \). In particular, we propose to define *relative fundamental uncertainty* as follows.

**Definition 1:**

Let \( \epsilon^* \equiv \frac{2\lambda+(1-\lambda)p}{2(p+\lambda-p\lambda)} \rho \) and \( \epsilon^{**} \equiv \frac{p+2(1-p)\lambda}{p} \rho \). Then,

- If \( \epsilon \leq \epsilon^* \), *relative fundamental uncertainty* is low,
- If \( \epsilon^* < \epsilon \leq \epsilon^{**} \), *relative fundamental uncertainty* is intermediate,
- If \( \epsilon > \epsilon^{**} \), *relative fundamental uncertainty* is high.

We use the label *relative* fundamental uncertainty because the ranges of the equilibria in this model depend not only on \( \epsilon \), which is a measure of fundamental uncertainty, but also on liquidity shocks and the probability of information asymmetry. Both boundary points, \( \epsilon^* \) and \( \epsilon^{**} \), increase with both the probability, \( \lambda \), and magnitude, \( \rho \), of liquidity shocks, while each decreases with the probability that the investor becomes privately informed. The relevance of the boundary points \( \epsilon^* \) and \( \epsilon^{**} \) can be attributed to shifts in investor behavior at these points, which in turn lead to discrete changes in the zero-profit bid quote, \( B \).

**Lemma 1:** In competitive equilibrium, with \( p > 0 \):

Conditional on incurring a liquidity shock, the investor sells if *relative fundamental uncertainty* is low, sells when she is informed with bad news or is uninformed if *relative fundamental uncertainty* is intermediate, and sells when she is informed with bad news if *relative fundamental uncertainty* is high. Conditional on not incurring a liquidity shock, the investor sells when she is informed with bad news, regardless of the level of *relative fundamental uncertainty*. This trading behavior supports bid quotes, \( B \), of
The bid quote is always discounted relative to the expected value of the asset, $\mu$, when $p$ is positive, to reflect the expected informational content of the sale. With high fundamental uncertainty, the investor sells in equilibrium whenever she is informed with bad news, while she retains her share if she is uninformed or informed with good news. The discount in the bid quote, $\epsilon \mu$, is the value of bad news.

With intermediate fundamental uncertainty the investor also sells in equilibrium in response to a liquidity shock when she is uninformed, and the possibility that the sale is uninformed allows a smaller discount,

$$\frac{p}{p+2(1-p)\lambda} \epsilon \mu.$$  

With low fundamental uncertainty the investor will sell in equilibrium upon incurring a liquidity shock, even if privately informed with good news, which supports a yet smaller discount,

$$\frac{(1-\lambda)p}{2\lambda+(1-\lambda)p} \epsilon \mu.$$  

The bid quote always decreases with fundamental uncertainty, $\epsilon$, and with the probability that the trader becomes privately informed, $p$, except when relative fundamental uncertainty is high (in which case the bid quote is already equal to the lowest possible asset value). The bid quote also increases with the probability of a liquidity shock, $\lambda$, except when fundamental uncertainty is high.

**IV.C. Firm Choice and Social Welfare with Competitive Markets**

We next assess the firm’s decision to conduct the IPO and the resulting social welfare with competitive market making. The firm will complete the IPO when $V_I$ exceeds the value of the assets to the firm, $V_F$, which given $M = 0$ requires $q \lambda \rho < \delta$. That is, the IPO occurs when the anticipated utility losses from secondary market illiquidity are sufficiently small relative to the firm’s need for cash.

**Proposition 1:** Given competitive market making and that the IPO price equals $V_I$:
(i) If relative fundamental uncertainty is low, the IPO occurs at the price $\mu$, and the improvement in social welfare is $W = \delta \mu$, the maximum attainable.

(ii) If relative fundamental uncertainty is intermediate, the IPO does not occur when $\delta < \frac{P}{2} \lambda \rho$. If the IPO occurs, the price is discounted to $V_I = (1 - \frac{P}{2} \lambda \rho) \mu$ and the improvement in social welfare is reduced to $W = \left( \delta - \frac{P}{2} \lambda \rho \right) \mu$.

(iii) If relative fundamental uncertainty is high, the IPO does not occur when $\delta < (1 - \frac{P}{2}) \lambda \rho$. If the IPO occurs, the price is further discounted to $V_I = (1 - \left(1 - \frac{P}{2}\right) \lambda \rho) \mu$, and the improvement in social welfare is further reduced to $W = \left( \delta - \left(1 - \frac{P}{2}\right) \lambda \rho \right)$.

Proposition 1 implies that competitive market making can lead to a non-discounted IPO and the largest possible improvement in welfare, but only when relative fundamental uncertainty is low. In contrast, markets will fail, either partially or completely, when relative fundamental uncertainty is intermediate or high. The range of parameters with market failure and the magnitude of the resulting welfare reduction are larger when relative fundamental uncertainty is high rather than intermediate. The market failure leads to pareto-dominated outcomes, as discussed in section IV.D.

Why The Competitive Market Can Fail.

The potential market failure with competitive market making is attributable to an information-based externality. Efficiency gains occur in this model when the asset is sold to a party who values it more highly, i.e. by the firm to the investor at $t=0$ or by the investor, should she suffer a liquidity shock, to the market makers at $t=1$. If the secondary market bid price is low enough that the investor will, in some states, be dissuaded from selling in the secondary market after a liquidity shock, then she will
reduce the amount she is willing to pay at the IPO.\textsuperscript{14} If the IPO price is reduced sufficiently the IPO does not occur.

The inefficiency arises because the competitive secondary market bid price is reduced to offset expected market maker losses attributable to the possibility that the trader is privately informed regarding the asset’s final value. A key point is that while trading losses due to asymmetric information are indeed a private cost from the viewpoint of a market maker, any loss to market makers is a gain to the investor, and the net social cost of informed trading is zero when aggregated across all agents. Since the private cost to market makers exceeds the social cost of completing trades, the competitive bid price is discounted by an amount greater than the net social cost of providing liquidity. Social welfare is damaged if this discounting of the bid price potentially dissuades the trader from completing a secondary market trade that would have enhanced welfare.

The inefficiency arises only when the discounting of the bid price is sufficient to dissuade trading in response to liquidity shocks, in at least some states of the world. If fundamental uncertainty and the likelihood of private information shocks are sufficiently low, the discounting of the bid price is modest and efficiency is not damaged. With greater fundamental uncertainty and/or a higher likelihood of information asymmetry the inefficiency becomes more severe, and can dissuade the firm from conducting the IPO.

\textbf{IV.D. The Role of a DMM Contract in a Competitive Market for Liquidity Provision}

The preceding discussion of how the competitive market can fail provides background for the role played by a DMM agreement. In particular, efficiency is enhanced by avoiding possible outcomes where the investor is dissuaded by a low bid price from selling in the secondary market after suffering a liquidity shock.

\textsuperscript{14} Note that the discounting of the IPO price that arises in our model differs from the widely-studied “underpricing” of IPOs. The latter refers to an IPO offer price that is less than the open market value of the share, while our model focuses an illiquidity-induced reduction in the open market value of the share itself.
Lemma 2: Setting the bid quote to \( B^* = (1 + \epsilon - \rho)\mu \) or higher is necessary and sufficient to ensure that \( q = 0 \), i.e. that the investor will always sell in the \( t=1 \) secondary market after suffering a liquidity shock.

Conditional on suffering a liquidity shock, the investor’s possible subjective valuations are, in increasing order, \((1 - \epsilon - \rho)\mu\) if she is informed that the asset value is low, \((1 - \rho)\mu\) if she is uninformed, and \((1 + \epsilon - \rho)\mu\) if she is informed that the asset value is high. Thus a bid quote of \( B^* \) or higher matches or exceeds the highest possible subjective valuation in the presence of a liquidity shock, implying that the investor will always sell conditional on a liquidity shock.\(^{15}\)

We now consider the effect of a potential DMM contract that calls for a flat \( t=0 \) payment, \( C \), from the firm to the DMM, in exchange for a commitment to maintain a minimum bid in the \( t=1 \) secondary market.\(^{16}\) The contracted increase in the bid price beyond the competitive level implies that the DMM will on average suffer monetary losses in trading with the investor.

Lemma 3: When relative fundamental uncertainty is intermediate or high and the bid price is set to \( B^* \), the DMM’s expected trading loss is:

\[
M(B^*) = \left( \frac{P}{2} (1 - \lambda) + \lambda \right) \rho \left( \frac{\epsilon}{\epsilon^*} - 1 \right) \mu > 0 \text{ if } \rho > \frac{\epsilon}{\epsilon^*} \text{ and }
\]

\[
M(B^*) = (\epsilon - \rho + (1 - \lambda) \frac{P}{2} \rho)\mu > 0 \text{ if } \rho < \frac{\epsilon}{\epsilon^*}.
\]

The firm will enter a DMM contract if its net gain \( V_I - V_F - C \) is larger with the contract than without, and is positive, which implies the following central proposition.

\(^{15}\) Note though that a bid quote of \( B^* \) does not ensure that the investor always sells, as the investor’s highest possible subjective valuation is \((1 + \epsilon)\mu > B^*\), which is attained if she is privately informed that the asset value is high, and does not suffer a liquidity shock. However, a sale motivated by information would create a market maker loss equivalent to the investor gain, with no net welfare gain.

\(^{16}\) Given that the firm values cash more highly than the investor it might appear efficient for the investor rather than the firm to contract with the DMM. In practice this alternative would lead to coordination issues across the multiple investors that typically participate in an IPO. Note also that in the present model investors effectively provide the cash that the firm uses to contract with the DMM, in the form of a higher IPO price.
Proposition 2: Given competitive market making and that the IPO price equals \( V \):

(i) If relative fundamental uncertainty is low the firm maximizes its value and also social welfare by completing the IPO without a DMM agreement.

(ii) If relative fundamental uncertainty is intermediate or high the firm maximizes its value and social welfare by completing the IPO and entering a DMM contract in which the firm pays the DMM the amount \( C = M(B^*) \), in exchange for a commitment to maintain the bid price \( B^* = (1 + \epsilon - \rho)\mu \).\(^{17}\)

Proposition 2 follows immediately from the preceding discussions. When relative fundamental uncertainty is low there is no market failure and no need for a DMM agreement. When relative fundamental uncertainty is intermediate or high the competitive market fails either partially or completely. However, the market failure can be cured by a DMM agreement that requires the bid price to be set high enough to ensure that the investor always sells following a liquidity shock. The DMM agreement leads to a Pareto improvement. The DMM is fully compensated in expectation for trading losses, the investor values the assets more highly and pays a correspondingly higher price to the firm, while the amount that the firm realizes in the IPO is increased by more than enough to pay the DMM.

*Why the Increase in Value Exceeds the Required Payment to the Market Maker*

From expression (1), anticipated monetary trading gains increase the \( t=0 \) value of the asset to the investor. However, increasing the bid quote also increases the value of the asset to the investor by the amount \( q\lambda\rho\mu \) as a consequence of reducing \( q \), the probability of not trading after a liquidity shock, to zero. That is, asset value is enhanced both because the investor enjoys monetary trading gains and because she avoids utility losses due to non-trading after liquidity shocks. The DMM needs to be compensated for

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\(^{17}\) Setting the contracted bid price higher than \( B^* \) would involve greater investor gains and market maker losses, a higher IPO price, and a larger required payment from the firm, but these increased payments would be zero sum.
the former, but not the latter. Since monetary trading gains are zero-sum, the increase in value net of the DMM payment is entirely attributable to the avoidance of outcomes where the investor is dissuaded from trading in response to a liquidity shock.

**Testable Implications and Discussion**

Several aspects of this analysis deserve emphasis. Most importantly, Proposition 2 presents testable implications regarding the conditions where DMM agreements are likely to be observed. Competitive liquidity provision is efficient if relative fundamental uncertainty is low, i.e. if $\epsilon < \epsilon^*$, while DMM agreements will be efficient if relative fundamental uncertainty is intermediate or high, i.e. if $\epsilon^* < \epsilon$. As previously noted, the boundary point $\epsilon^*$ increases with the probability, $\lambda$, and magnitude, $\rho$, of investor liquidity shocks, implying that larger and more frequent investor liquidity shocks reduce the need for DMMs, *ceteris paribus*. However, these liquidity shock parameters are investor characteristics that do not differ across firms if investors hold diversified portfolios.

The breakpoint $\epsilon^*$ decreases with $p$, the probability that the investor will become informed. This implies that DMM agreements will be value enhancing (i) for those firms and/or at those times when uncertainty regarding asset value, $\epsilon$, is high, in combination with (ii) the probability that private information is received, $p$, is high. The former arises from the fact that our model implies that DMM contracts enhance value when $\epsilon > \epsilon^*$, while the latter arises because the threshold $\epsilon^*$ is lower for firms or at times when $p$ is greater. The model therefore implies that DMM contracts are most likely to be value enhancing for smaller and younger firms, to the extent these are characterized by more fundamental uncertainty and greater information asymmetries. This implication is generally consistent with the empirical evidence, e.g. Venkataraman and Waisburd (2007), Anand, Tanggaard, and Weaver (2009), and Menkveld and Wang (2013).

The model further implies that DMM agreements that are voluntarily adopted should be observed more frequently for firms where empirical measures of information asymmetry (e.g. the “PIN” measure
attributable to Easley, Kiefer, O’Hara and Paperman (1996) or the price impact measure of Huang and Stoll (1996)) and fundamental volatility are large in combination. Further, in light of our prediction that firms with high relative fundamental uncertainty will self-select to enter DMM agreements, future researchers need to consider the resulting endogeneity when interpreting any observed cross-sectional correlation between the existence of DMM agreements and empirical measures such as PIN or return volatility. In contrast, should DMM agreements ever be mandated based on an arbitrary threshold such as firm size then an appropriate research design (e.g. a regression discontinuity approach) may be able to estimate causal effects of the DMM agreement on market outcomes.

In addition to altering the breakpoint, $\epsilon^*$, that delineates the range of parameters where a DMM contract is efficient, changes in the probability that private information is received, $p$, affect both the magnitude of the efficiency gain from a DMM contract and the size of the requisite payment to the market maker. Regarding the latter, Lemma 3 also implies testable implications. When a DMM contract is efficient, the required payment to the market maker is strictly increasing in uncertainty regarding fundamental value, $\epsilon$, and in the probability that the trader is privately informed, $p$.

Our model considers explicitly only liquidity shocks that reduce the investor’s subjective valuation and induce a desire to sell. Of course investors may also experience liquidity shocks (e.g. a sudden cash inflow) that induce a desire to buy. The direct extension to a model with both positive and negative liquidity shocks is a contract calling for the DMM to both decrease the ask price and increase the bid price, i.e. to narrow the bid-ask spread, relative to competitive levels. As noted, DMM contracts that require the bid-ask spread to be kept within a narrow range are in fact observed on a number of markets, and are typically employed for less liquid stocks. Note, though, that our model provides the policy implication that DMM contracts will enhance value and efficiency only when used to offset market maker trading costs attributable to the information content of trades. Value is not enhanced by constraining bid-ask spreads to be narrower than the social cost (e.g. inventory-carrying costs or order-processing costs) of completing trades, which may also be greater for thinly-traded securities.
Our model also helps to rationalize the empirical observation (e.g. Schultz and Zaman, 1994 and Ellis, Michaely, and O’Hara, 2000) that underwriters often act to support or stabilize post-IPO market prices. Such aftermarket support potentially complements a DMM contract, focusing in particular on the period immediately after the IPO when information asymmetries are potentially large. However, such stabilization activities are typically temporary, and therefore do not comprise a complete substitute for longer-term DMM contracts.

If the probability of informed trading is perceived by market participants to vary across time, relative fundamental uncertainty is dynamic. A firm that is typically characterized by low relative fundamental uncertainty may not always be so. While the model presented here is static, the economic reasoning suggests that in addition to contracts that require DMMs to routinely narrow spreads for smaller and younger firms, DMM contracts that impose a binding obligation during times of increased asymmetric information may be welfare and value enhancing for larger and better known firms. To implement such a contractual obligation it would be useful to develop methods of detecting, on the basis of observable market data, periods of heightened information asymmetry. One simple approach could be to contract for a maximum bid-ask spread that is sufficiently greater than the typical spread for the stock that the contract would be unlikely to bind except at times of heightened information asymmetry.


The preceding section demonstrates that competitive market making need not lead to efficient outcomes, as the market may fail partially or fully when relative fundamental uncertainty exceeds a threshold. However, if the firm has complete bargaining power, in the sense that it captures in a higher IPO price any increase in the value of the asset attributable to better secondary market liquidity, then a value-maximizing firm will cure any market failure by contracting with a market maker to enhance secondary market liquidity.

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18 See for example Easley, Lopez de Prado, and O’Hara (2012) who present one such measure.
V.A. Partial Bargaining Power in the IPO Market

In this section, we assess the effects of relaxing the assumption that the firm captures all of the benefits from enhanced liquidity. In practice, firms’ proceeds from IPOs are typically less than the open market value of the shares issued, reflecting that some of the gains from trade are captured by intermediaries in the form of commissions and by investors in the form of the widely-studied “underpricing” of shares. A full model of the IPO process is beyond the scope of this paper. We instead assume that the bargaining game generates a parameter, $\beta$, which is the proportion of the improvement in $t=0$ value that is captured by the firm. In particular, the IPO price is $V_F + \beta(V_I - V_F)$. Stated alternatively, the improvement in firm value from completing the IPO, net of a possible payment to the DMM, $C$, is

$$\pi = \beta(V_I - V_F) - C. \quad (3)$$

Using expression (1) along with the definition of $V_F$ in expression (3), the firm’s net gain can also be expressed as

$$\pi = \beta\mu(\delta - q(B)\lambda \rho) + \beta M(B) - C. \quad (4)$$

If the firm conducts the IPO in the competitive market and declines to enter a DMM agreement ($C = M = 0$) then the firm’s net gain is $\pi = \beta\mu(\delta - q(B)\lambda \rho)$. The model therefore implies the following:

Lemma 4: The market will not fail if relative fundamental uncertainty is low, for any $\beta > 0$.

Lemma 4 follows from the prior observation that when relative fundamental uncertainty is low competitive market making without a DMM agreement leads to a bid price high enough that the investor

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19 See, for example, Chen and Ritter (2000) for analysis of investment bank commissions and Ibbotson, Sindelar, and Ritter (1994) for evidence regarding underpricing.
always sells in response to a liquidity shock, which means \( q(B) = 0 \) and that the firm’s net gain is \( \pi = \beta \delta \mu \). The firm will therefore conduct the IPO for any \( \beta > 0 \), i.e. as long as it captures any positive portion of the benefit from doing so. Of course, this result would be altered if there was any fixed cost to completing the IPO, in which case the firm would decline to proceed with the IPO for sufficiently small \( \beta \), even with low relative fundamental uncertainty.

If the firm conducts the IPO and enters a DMM agreement to set the bid price to \( B^* \) in exchange for a fixed payment \( C^* = M(B^*) \), then \( q(B^*) = 0 \) and, from expression (4), the firm’s net gain is \( \pi = \beta \mu \delta - C^* (1 - \beta) \).

The market will fail completely if the firm’s net gain is negative with or without a DMM agreement, while the market will fail partially if the firm’s net gain is positive without a DMM agreement and is smaller with the optimal DMM agreement than without. This implies that, with intermediate or high relative fundamental uncertainty, the market will fail completely if

\[
\delta < q(B) \lambda \rho \quad \text{and} \quad \beta < \frac{C^*}{C^* \mu + \delta},
\]

while the market will fail partially if

\[
\delta \geq q(B) \lambda \rho \quad \text{and} \quad \beta < \frac{C^*}{C^* + q(B) \lambda \rho \mu}.
\]

Market failure occurs when the firm’s bargaining power in the IPO market, \( \beta \), is less than a critical level that depends on the payment to the market maker, \( C^* \). Whether the potential market failure is partial or complete depends on the firm’s need for cash, \( \delta \), relative to a critical level that depends on the probability, \( q \), that the investor does not sell after suffering a liquidity shock.

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\(^{20}\) We show in the Appendix that when \( \beta < 1 \) and relative fundamental uncertainty is high the firm may, for some parameters, prefer a DMM contract calling for a bid price lower than welfare maximizing level, \( B^* \). As the general intuition of the firm’s decision is unchanged, we continue for simplicity to focus here on a DMM contract calling for the welfare maximizing bid price.
The critical levels of $\delta$ and $\beta$ can be stated in terms of the model’s exogenous parameters, as summarized in Table 2.

**Table 2: Key thresholds levels of $\delta$ and $\beta$.**

<table>
<thead>
<tr>
<th>Range of $\epsilon$</th>
<th>$\delta^*$</th>
<th>$\beta^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\epsilon^* &lt; \epsilon \leq \rho$</td>
<td>$\delta^* = \frac{\lambda P}{2} \rho$</td>
<td>When $\delta &lt; \delta^<em>$, $\beta^</em> = \frac{e(p + \lambda - \rho) - \rho \left( \frac{1}{2}p + \lambda - \frac{1}{2} \rho \right)}{\delta + e(p + \lambda - \rho) - \rho \left( \frac{1}{2}p + \lambda - \frac{1}{2} \rho \right)}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>When $\delta \geq \delta^<em>$, $\beta^</em> = \frac{e(p + \lambda - \rho) - \rho \left( \frac{1}{2}p + \lambda - \frac{1}{2} \rho \right)}{\lambda \rho + e(p + \lambda - \rho) - \rho \left( \frac{1}{2}p + \lambda - \frac{1}{2} \rho \right)}$</td>
</tr>
<tr>
<td>$\rho &lt; \epsilon \leq \epsilon^{**}$</td>
<td>$\delta^* = \frac{\lambda P}{2} \rho$</td>
<td>When $\delta &lt; \delta^<em>$, $\beta^</em> = \frac{e-p+(1-\lambda)e}{\delta+e-p+(1-\lambda)e} \frac{\rho}{2}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>When $\delta \geq \delta^<em>$, $\beta^</em> = \frac{e-p+(1-\lambda)e}{\lambda \rho + e-p+(1-\lambda)e} \frac{\rho}{2}$</td>
</tr>
<tr>
<td>$\epsilon &gt; \epsilon^{**}$</td>
<td>$\delta^* = \left( 1 - \frac{P}{2} \right) \lambda \rho$</td>
<td>When $\delta &lt; \delta^<em>$, $\beta^</em> = \frac{e-p+(1-\lambda)e}{\delta+e-p+(1-\lambda)e} \frac{\rho}{2}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>When $\delta \geq \delta^<em>$, $\beta^</em> = \frac{e-p+(1-\lambda)e}{e-p+(1-\lambda)e+\lambda \rho + \lambda(1-p) \rho}$</td>
</tr>
</tbody>
</table>

These discussions and definitions allow us to state the following.

**Proposition 3:** Given competitive market making and the possibility of entering a DMM agreement:

(i) When relative fundamental uncertainty is intermediate or high the firm completes the IPO and enters the optimal DMM agreement only if the firm’s bargaining power exceeds the threshold level, $\beta^*$. When $\beta < \beta^*$ the market fails. The failure is partial, as the firm completes the IPO but does not enter the DMM contract, when $\delta$ exceeds the threshold level, $\delta^*$, while the failure is complete otherwise.

(ii) The threshold level of bargaining power, $\beta^*$, decreases with the probability $\lambda$ and magnitude $\rho$ of investor liquidity shocks, and for most parameter values increases with the probability that private information is received.

The intuition for Proposition 3 is simply that the firm bears the full cost of compensating the DMM, while it captures only the proportion $\beta$ of the resulting increase in value. The market fails if the
firm’s bargaining power, $\beta$, is sufficiently low. The failure is partial if the firm’s need for cash is sufficiently strong, and is total otherwise.

We show in the appendix that the threshold level $\beta^*$ decreases, implying that efficient outcomes are obtained for a broader range of $\beta$, when both the probability and magnitude of investor liquidity shocks are greater. This reflects that more liquidity-motivated trading leads to smaller expected trading losses and a smaller required payment to market makers who have contracted to maintain a bid quote equal to $B^*$. Also, for most parameter values the firm’s net gain with a DMM contract decreases with $p$, the probability that the trader becomes privately informed, due to greater expected market maker trading losses which must be fully compensated by the firm. Thus higher $p$ leads to larger $\beta^*$ and market failure occurs over a broader range of bargaining parameters.\textsuperscript{21}

Chen and Ritter (2000) argue that investment bankers earn economic rents from conducting IPOs. Our analysis indicates that frictions such as investment bank commissions or market underpricing, which prevent firms from capturing the full market value of the assets they sell, can cause complete market failure in the sense that firms decline to proceed with efficiency-enhancing IPOs. Our model also produces the more subtle result that such frictions can lead to a partial market failure, where a firm with a strong need for cash conducts the IPO, but does not enter the efficient DMM contract. Market failure due to reduced firm market power are less likely if investors are more likely to be exposed to large liquidity shocks, but are more likely for firms with a higher likelihood that of information asymmetry.

Outcomes characterized by full or partial market failure attributable to the firm’s lack of bargaining power are Pareto inefficient. Outcomes could be improved if the investor (or in more realistic settings, IPO investors acting collectively) were to sponsor a DMM. As noted, after-market support provided by underwriting firms can be interpreted to effectively reflect a short term DMM

\textsuperscript{21} The exception arises in the range where $\rho < \epsilon \leq \epsilon^\prime$ and $\delta \geq \lambda \frac{B}{\sigma} \rho$. Here, the net gain from the IPO is positive without a DMM contract, so the DMM decision depends on the comparison between net gains with and without the DMM contract. In this range the net gain to the firm decreases faster without than with the DMM contract, leading $\beta^*$ to decrease with $p$. 

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function. Our model implies that an implicit or formal agreement by which investors compensate the underwriter for enhancing secondary market liquidity can also be efficient and value enhancing.

Proposition 3 has the testable implication that firms with stronger bargaining power in the IPO process will be more likely to complete IPOs and to enter DMM agreements when efficient. For example, Jeon, Lee, Nasser and Via (2013) suggest that IPO firms backed by private equity or venture capital firms have greater negotiating power (higher $\beta$) because these firms have well-informed principals and repeat players in the IPO market. Burch, Nanda, and Warther (2005) argue that investment banks with larger shares of IPO deals in an industry have a stronger bargaining position, implying lower firm $\beta$, while firms with larger offerings have greater negotiating power.

V.B. The Effect of Possible Monopoly Power in Liquidity Provision

To this point, we have assumed that market makers engage in Bertrand competition, leading to zero expected market making profits in the absence of a DMM contract. However, the business of liquidity provision could be imperfectly competitive. Glosten (1989) models the case of a monopolist liquidity provider, and the model presented by Bernhardt and Hughson (1997) allows for positive expected market making profits in equilibrium. In Biais, Martimort, and Rochet (2000) each liquidity supplier acts to maximize profits while taking as fixed the behavior of competitors, and economic profits persist in equilibrium as long as the number of competitors is finite. Further, depending on the degree of cross-market competition, a trading exchange may be able to extract monopoly rents in the form of trading fees, even if competition prevails among potential liquidity suppliers on the exchange. Monopoly rents may also arise if the tick size is large enough to constrain bid-ask spreads to be larger than the competitive level.

A full model of imperfect competition in market making is beyond the scope of this paper. To obtain insights into the potential role of market maker rents that are as general as possible and in the simplest possible manner, we assume that market makers are able to extract an exogenously specified
level of trading profits in the absence of a DMM agreement. We also assume that the potential DMM agreement would call for a bid price equal to $B^*$, in exchange for a payment from the firm to the DMM that is sufficient to compensate the DMM both for their anticipated trading losses as specified in Lemma 2 and for foregone market making rents. Let $B'$ denote the secondary market bid price given market power, which is less than the competitive bid, and let $M(B')$ denote the trader’s expected trading profit in the absence of a DMM agreement (so that $-M(B')$ is the market making rent). The payment $C$ from the firm to the DMM in exchange for maintaining a bid price equal to $B^*$ is $M(B^*) - M(B')$.\(^\text{22}\)

**Proposition 4**: Given the existence of rents in market making:

(i) In contrast to outcomes with competitive market making, the market can fail completely as the firm declines to complete the IPO, even if relative fundamental uncertainty is low and $\beta = 1$.

(ii) Larger market making rents strictly increase the range of parameters that lead to market failure.

Monopoly rents in market making affect outcomes in a manner that is similar to the effects of asymmetric information. Each leads to a reduction in the secondary market bid price relative to the fundamental value of the asset, $\mu$. Monopoly rents, like market making losses due to asymmetric information, are zero-sum transfers, not social costs of providing liquidity. Social welfare is reduced by market making rents in the same manner as information asymmetry, i.e. if the market making rents lead to bid prices low enough that the investor may be dissuaded from selling the asset after a liquidity shock.

However, the effects of market making returns are not identical to those of asymmetric information, as the former can lead to market failure even when relative fundamental uncertainty is low,\(^\text{22}\) in this setting, market makers are able to fully preserve the existing market making rents upon entering a DMM agreement. More broadly, the qualitative predictions of Proposition 4 will continue to hold if market makers retain any portion of their market making rents. If market makers are not able to preserve any of their rents, i.e., if the firm’s bargaining power versus market makers is complete, then $C$ can be set to $M(B^*)$ and outcomes with a potential DMM agreement are identical to the competitive secondary market case.
$\beta = 1$, and the firm has the option to enter a DMM contract, while the latter cannot. Complete market failure occurs if the firm’s net gains are negative with or without a DMM agreement. Given $M(B') < 0$, the market fails completely if

$$\delta < q(B')\lambda \rho - M(B')/\mu \quad \text{and} \quad \beta < \frac{M(B^*) - M(B')}{M(B^*) + q(B')\lambda \rho \mu}.$$ 

The first of these conditions implies that the firm net gain is negative if the firm completes the IPO without entering a DMM agreement. This condition can be met even when relative fundamental uncertainty is low, because the investor reduces the price she is willing to pay for the assets in anticipation of trading losses attributable to market maker rents, $-M(B')$, and because the bid price $B'$ set in the presence of market making rents is not necessarily high enough to ensure $q(B') = 0$, i.e. that the investor always sells in response to a liquidity shock. The second condition implies that the firm net gain is negative with the optimal DMM agreement. Even with $\beta = 1$ this condition can be met if $M(B') < -\mu \delta$. Further, increased market maker rents strictly increase the range of $\beta$ over which the IPO does not occur.

Partial market failure occurs if the firm completes the IPO, i.e. $\delta > (q(B')\lambda \rho - M(B')/\mu)$, but the firm net gain is lower with the optimal DMM agreement than without. The latter occurs if

$$\beta < \frac{M(B^*) - M(B')}{M(B^*) - M(B') + q(B')\lambda \rho \mu}.$$ 

Since the right hand side of this expression increases with larger market maker rents, the model also implies that partial market failure occurs for a broader range of firm bargaining power when market making rents are greater.

An advisory panel to the U.S. SEC has recently recommended increases in the tick size for smaller exchange-traded companies. The intent appears to be to increase bid-ask spreads beyond competitive levels in hopes that the resulting market making rents will attract additional liquidity.
suppliers, who in turn may provide other services such as research reports. However, our model indicates that such a proposal will likely be counterproductive. In particular the model implies that monopoly profits in secondary market liquidity provision damage efficiency and reduce IPO prices, as investors take into account the higher costs of secondary market trading. Monopoly rents in secondary market liquidity provision can lead to complete market failure, where the firm chooses to not complete the IPO. In contrast, DMM contracts of the type considered here improve efficiency and facilitate the IPO process.

VI. Conclusions and Extensions

We present a relatively simple model to assess the effects on firm value and social welfare of a contract by which a firm engages a Designated Market Maker (DMM) to improve secondary market liquidity. We show that a DMM contract can improve value and welfare because of an information-based externality. In particular, market maker losses from transactions with privately-informed traders are a private cost of providing liquidity, but comprise a zero-sum transfer rather than a cost when aggregated across all agents. Since the private costs of providing liquidity exceed the social costs, a competitive market provides less liquidity than the efficient level. DMM contracts comprise a potential market solution to this market imperfection. While contracts of the type modeled here are observed on some international markets, they are currently prohibited in the U.S. by FINRA Rule 5250.

Our analysis shows that the potential benefits of contracts that require enhanced liquidity supply are related to information, rather than illiquidity, per se. Our model’s implications differ in an important but subtle way from the simple insight that enhancing liquidity is useful in otherwise illiquid stocks. If stocks are illiquid due to high real costs of competing trades, e.g. due to the inventory costs that Demsetz (1968) predicts will be high for thinly-traded assets, then the marginal social cost of providing liquidity is high, and it is efficient for spreads to be wide. Value will not be enhanced by contracts that reduce spreads below the real social costs of providing liquidity. Instead, our analysis implies that efficiency and firm value can be enhanced for those firms or at those times when markets are illiquid due to a
combination of high fundamental uncertainty and information asymmetries. In the cross section, our analysis implies that DMM contracts are likely to be useful for smaller and younger firms, as opposed to larger firms with a high proportion of assets-in-place. In the time series, our analysis implies that DMM contracts may be useful if they require additional liquidity provision at times when perceived fundamental uncertainty and information asymmetries are temporarily elevated.

The model presented here has a number of implications for researchers and policy makers. First, and most important, our analysis shows that economic efficiency and firm value can be enhanced by contracts that require liquidity providers to supply more liquidity than they would otherwise choose. In light of the absence of barriers to entry in providing liquidity in electronic limit order markets, the business of liquidity provision may well reflect competitive equilibrium. If so, side payments will be required to induce one or more designated market makers to take on such affirmative obligations, and our model implies that affirmative obligations to provide liquidity should not be imposed in the absence of compensation to the DMMs. Our model also indicates that recent proposals to increase the minimum price increment for the trading of smaller stocks in order to attract additional liquidity providers and facilitate IPOs are also likely to be counterproductive, as investors will take into account wider bid-ask spreads in valuing shares. In contrast, contracts of the type studied here, calling for a contract between the listed firm and the DMM, enhance liquidity, improve firm value, and facilitate IPOs. Our analysis also supports the intuition that IPOs can be facilitated by improving firm’s bargaining power in the IPO process, e.g. by measures that intensify competition among investment banks.

Several limitations of our analysis, each of which provides useful opportunities for future research, should be noted. Our model focuses for simplicity on a single investor, a single round of secondary trading, and trades of fixed size. As such, we have not considered potential effects of market maker affirmative obligations on trade timing, trade sizes, repeat trading, or trading aggressiveness.
We also assume that the investor either transacts at the \( t=1 \) market-maker quote or does not trade, without allowing the trader the option to leave a limit order on the book. In a model extended to allow for multiple rounds of trading the trader might prefer to enter a limit order. While a formal analysis of a dynamic limit order book in the presence of a DMM is beyond the scope of this paper, we offer the following conjectures. First, as in Sabourin (2006), the introduction of a DMM will cause some traders who would have entered a limit order to submit a market order instead. Second, the \( t=1 \) seller would enter a limit order instead of transacting at the market bid quote only when her utility was increased by doing so. As a consequence, a contract calling for a given bid quote in a limit order market cannot be less valuable to the investor as compared to the same requirement in the absence of the option to enter limit orders. If the trader’s option to enter a limit order benefits or at least does not harm the DMM, then the key implication of this model, that value and welfare are enhanced by a DMM agreement will survive. Third, the \( t=1 \) seller would not consider entering a limit order unless it is sufficiently probable that another trader with a private valuation in excess of the \( t=1 \) bid will subsequently arrive. As in Kandel and Liu (2006), private information could motivate the use of a limit order. If the \( t=1 \) trader is privately informed that the asset value is high then she can be more confident that the subsequent trader (who could also be informed) would also have a high valuation, and she will be more likely to enter a limit order with a price above the market maker’s bid instead of selling to the market maker. If so, the market maker’s adverse selection risk can be reduced by the trader’s option to enter the limit order, potentially leading to a lower required DMM payment in equilibrium.

We have not considered the full range of potential DMM agreements. We focus for simplicity on a contract by which the listed firm pays a flat fee to a DMM to increase the market bid price, or by extension, to narrow the bid-ask spread. Anand, Tanggaard and Weaver (2009) document two additional features of observed DMM contracts. First, DMM contracts in some cases call for a variable fee proportionate to the volume of trades accommodated, in addition to a fixed fee. Second, the obligation to narrow the spread can apply only a proportion (e.g. 90%) of the time. In the present model, the former
modification is irrelevant. This reflects that, given risk neutrality, any combination of fixed and variable payments that are equal in expectation to market maker trading losses is equally satisfactory. The latter modification, implying that the DMM will with some probability not enhance secondary market liquidity, would potentially be welfare reducing in the present model. These features of actual DMM agreements likely reflect capital limitations, which are not considered in our model. A variable fee tied to the volume of DMM transactions can be viewed as a form of risk sharing, by which a portion of trading losses that are, ex post, larger than anticipated are passed from the DMM to the firm. Similarly, the option to temporarily cease enhancing liquidity will reduce the magnitude of market maker losses during extreme events. Each provision therefore reduces the likelihood that the DMM will fail because it has exhausted its finite capital reserves.
References


Appendix:

Proof of Lemma 1:

There are six possible outcomes for the investor's $t=1$ subjective valuation of the asset, as shown in Figure 1.

![Diagram of investor's subjective valuation](image)

<table>
<thead>
<tr>
<th>Case</th>
<th>Investor's Subjective Valuation</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1)</td>
<td>$\mu(1 + \epsilon - \rho)$</td>
</tr>
<tr>
<td>(2)</td>
<td>$\mu(1 - \epsilon - \rho)$</td>
</tr>
<tr>
<td>(3)</td>
<td>$\mu(1 - \rho)$</td>
</tr>
<tr>
<td>(4)</td>
<td>$\mu(1 + \epsilon)$</td>
</tr>
<tr>
<td>(5)</td>
<td>$\mu(1 - \epsilon)$</td>
</tr>
<tr>
<td>(6)</td>
<td>$\mu$</td>
</tr>
</tbody>
</table>

Figure 1: The subjective valuation of the asset to the investor at $t=1$.

As in Glosten and Milgrom (1985) the competitive bid price equals the expected asset value conditional on a sale, implying zero expected profit. In this model the competitive bid price must lie in the range $[(1 - \epsilon)\mu, \mu]$. To see this, suppose first the bid price is weakly larger than $(1 + \epsilon)\mu$. The investor will sell in all six cases displayed in Figure 1, so the conditional expected asset value is $\mu$. A bid price lower than $(1 + \epsilon)\mu$ will lead to a conditional expectation of asset value less than $\mu$, because the investor with positive private news does not sell. This implies that the maximum asset value conditional on a sale is $\mu$, obtained only when the bid price is $(1 + \epsilon)\mu$. Therefore, a bid price weakly larger than $\mu$ leads to negative expected profit to the market maker. Alternatively, suppose the bid price is less than $(1 - \epsilon)\mu$. The investor only sells in Case (2), leading to a conditional expected asset value of $(1 - \epsilon)\mu$. This gives the market makers a positive expected profit and implies that the competitive bid quote will be
weakly larger than \((1 - \epsilon)\mu\). We therefore need only consider potential competitive bid prices, \(B\), within \([ (1 - \epsilon)\mu, \mu]\).

To assess possible competitive bid prices, we consider in turn three ranges of \(\epsilon\): (1) \(\epsilon \leq \epsilon^* (\epsilon^* \equiv \frac{2\lambda + (1 - \lambda)p}{2(p + \lambda - p\lambda)} \rho)\), (2) \(\epsilon^* < \epsilon \leq \epsilon^{**} (\epsilon^{**} \equiv \frac{p + 2(1 - p)\lambda}{p} \rho)\), and (3) \(\epsilon^{**} < \epsilon\).

Range (1):

When \(\epsilon \leq \epsilon^*\), let \(B = \left(1 - \frac{(1 - \lambda)p}{2\lambda + (1 - \lambda)p} \epsilon\right) \mu\). Note that \(B = \left(1 - \frac{(1 - \lambda)p}{2\lambda + (1 - \lambda)p} \epsilon\right) \mu = \left(1 + \epsilon - \frac{2(p + \lambda - p\lambda)}{2\lambda + (1 - \lambda)p} \mu \geq (1 + \epsilon - \rho)\mu\) because \(\epsilon \leq \frac{2\lambda + (1 - \lambda)p}{2(p + \lambda - p\lambda)} \rho\). With this bid price, the investor sells in Cases (1), (2), (3), and (5). Given this trading behavior, the conditional expected asset value is \((1 - \epsilon)\mu\), implying that \(B = \left(1 - \frac{(1 - \lambda)p}{2\lambda + (1 - \lambda)p} \epsilon\right) \mu\) is a possible competitive bid price.

Next, we will show that, when \(\epsilon \leq \epsilon^* < \rho\), \(B = \left(1 - \frac{(1 - \lambda)p}{2\lambda + (1 - \lambda)p} \epsilon\right) \mu\) is the unique competitive bid price. We do so by separating the range of \((0, \epsilon^*]\) into three sub-ranges, \((0, \frac{\rho}{2}]\), \(\left(\frac{\rho}{2}, \frac{2\lambda + p - 2p\lambda}{2(p + \lambda - p\lambda)} \rho\right]\) and \(\left(\frac{2\lambda + p - 2p\lambda}{2(p + \lambda - p\lambda)} \rho, \epsilon^*\right]\). In the first sub-range, any bid price within \([ (1 - \epsilon)\mu, \mu]\) is weakly larger than \((1 + \epsilon - \rho)\mu\) because \(\epsilon \leq \frac{\rho}{2}\). Therefore, the investor sells with any bid price within \([ (1 - \epsilon)\mu, \mu]\) in Cases (1), (2), (3), and (5), implying a conditional expected asset value of \((1 - \epsilon)\mu\). If \(B > \left(1 - \frac{(1 - \lambda)p}{2\lambda + (1 - \lambda)p} \epsilon\right) \mu\), the expected profit to the market makers is negative. In contrast, if \(B < \left(1 - \frac{(1 - \lambda)p}{2\lambda + (1 - \lambda)p} \epsilon\right) \mu\), the expected profit to the market makers is positive. Therefore, \(\left(1 - \frac{(1 - \lambda)p}{2\lambda + (1 - \lambda)p} \epsilon\right) \mu\) is the unique competitive bid price in the first sub-range.

In the second sub-range, because \(\frac{\rho}{2} < \epsilon \leq \frac{2\lambda + p - 2p\lambda}{2(p + \lambda - p\lambda)} \rho \leq (1 - \epsilon)\mu < (1 + \epsilon - \rho)\mu \leq \mu\) and \((1 - \rho)\mu < (1 - \epsilon)\mu\). So, \((1 + \epsilon - \rho)\mu\) is the only possible subjective valuation of the asset within
\[(1 - \epsilon)\mu, \mu \). If \( B \geq (1 + \epsilon - \rho)\mu \), the investor sells in Cases (1), (2), (3), and (5). This means the conditional expected asset value is \( 1 - \frac{(1-\lambda)p}{2\lambda+(1-\lambda)p} \epsilon \mu \), which is greater than \( (1 + \epsilon - \rho)\mu \), as noted earlier. Therefore, \( 1 - \frac{(1-\lambda)p}{2\lambda+(1-\lambda)p} \epsilon \mu \) is the only possible competitive bid price that is weakly larger than \( (1 + \epsilon - \rho)\mu \) and less than \( \mu \). Suppose \( B < (1 + \epsilon - \rho)\mu \). Since \( \epsilon < \rho \) implies \( (1 - \rho)\mu < (1 - \epsilon)\mu \), the investor sells in Cases (2), (3), and (5). In this case, the conditional expected asset value is \( 1 - \frac{(1-\lambda)p}{2\lambda+(1-\lambda)p} \epsilon \mu \), which is weakly larger than \( (1 + \epsilon - \rho)\mu \), as noted earlier. Therefore, \( 1 - \frac{(1-\lambda)p}{2\lambda+(1-\lambda)p} \epsilon \mu \) is the only possible competitive bid price that is weakly larger than \( (1 + \epsilon - \rho)\mu \) and less than \( \mu \). Suppose \( \epsilon \geq (1 + \epsilon - \rho)\mu \). Since \( \epsilon \geq \rho \) implies \( (1 - \rho)\mu \leq (1 + \epsilon)\mu \), the investor sells in Cases (2), (3), and (5). In this case, the conditional expected asset value is \( 1 - \frac{p}{p+2(1-p)\lambda} \epsilon \mu \), which is weakly larger than \( (1 + \epsilon - \rho)\mu \), as noted earlier. Therefore, \( 1 - \frac{p}{p+2(1-p)\lambda} \epsilon \mu \) is the only possible competitive bid price that is weakly larger than \( (1 + \epsilon - \rho)\mu \) and less than \( \mu \). However, there is a bid price less than \( (1 + \epsilon - \rho)\mu \) that can also meet the zero-expected profit condition. When \( B < (1 + \epsilon - \rho)\mu \), since \( \epsilon < \rho \) implies \( (1 - \rho)\mu < (1 - \epsilon)\mu \), the investor sells in Cases (2), (3), and (5), which leads to a conditional expected asset value \( 1 - \frac{p}{p+2(1-p)\lambda} \epsilon \mu \). Because \( 1 - \frac{p}{p+2(1-p)\lambda} \epsilon \mu = \left( 1 + \frac{p+2(1-p)\lambda}{p\lambda} \epsilon \right) \mu = \left( 1 + \epsilon - \frac{2p+2(1-p)\lambda}{p\lambda} \epsilon \right) \mu \), it is the only possible competitive bid price when \( \epsilon \leq \frac{2\lambda+p-2p\lambda}{2(p+\lambda-p\lambda)} \rho < \epsilon^* \).

In the third sub-range, with \( \frac{p}{2} < \frac{2\lambda+p-2p\lambda}{2(p+\lambda-p\lambda)} \rho < \epsilon < \epsilon^* < \rho \), \( 1 - \frac{(1-\lambda)p}{2\lambda+(1-\lambda)p} \epsilon \mu \) is the only possible competitive bid price that is weakly larger than \( (1 + \epsilon - \rho)\mu \) and less than \( \mu \). However, there is a bid price less than \( (1 + \epsilon - \rho)\mu \) that can also meet the zero-expected profit condition. When \( B < (1 + \epsilon - \rho)\mu \), since \( \epsilon < \rho \) implies \( (1 - \rho)\mu < (1 - \epsilon)\mu \), the investor sells in Cases (2), (3), and (5), which leads to a conditional expected asset value \( 1 - \frac{p}{p+2(1-p)\lambda} \epsilon \mu \). Because \( 1 - \frac{p}{p+2(1-p)\lambda} \epsilon \mu = \left( 1 + \epsilon - \frac{p+2(1-p)\lambda}{p\lambda} \epsilon \right) \mu = \left( 1 + \epsilon - \frac{2p+2(1-p)\lambda}{p\lambda} \epsilon \right) \mu < \left( 1 + \epsilon - \frac{2p+2(1-p)\lambda}{p\lambda} \epsilon \right) \mu \), it also satisfies the necessary condition for a competitive bid price. We assume that competition between market makers leads to the higher of the two zero-expected profit bid prices in equilibrium, i.e. \( B = (1 - \frac{(1-\lambda)p}{2\lambda+(1-\lambda)p} \epsilon)\mu \).
Range (2):

When $\epsilon^* < \epsilon \leq \epsilon^*$, let $B = \left(1 - \frac{p}{p + 2(1-\rho)\lambda} \epsilon\right)$, $B = \left(1 + \epsilon - \frac{p}{p + 2(1-\rho)\lambda} \epsilon\right) = \left(1 + \epsilon - \frac{2p + 2\lambda - 2p\lambda}{p + 2(1-\rho)\lambda} \epsilon\right) \mu < (1 + \epsilon - \rho) \mu$ because $\epsilon^* < \epsilon$. And $B = \left(1 - \frac{p}{p + 2(1-\rho)\lambda} \epsilon\right) \mu \geq (1 - \rho) \mu$ because $\epsilon \leq \epsilon^*$. With this bid price, the investor sells in Cases (2), (3), and (5), implying a conditional expected asset value of $\left(1 - \frac{p}{p + 2(1-\rho)\lambda} \epsilon\right) \mu$. So $B = \left(1 - \frac{p}{p + 2(1-\rho)\lambda} \epsilon\right) \mu$ is a possible competitive bid price.

Next, we will show that, when $\epsilon^* < \epsilon \leq \epsilon^*$, $B = \left(1 - \frac{p}{p + 2(1-\rho)\lambda} \epsilon\right) \mu$ is the unique competitive bid price. To do so we separate the range $((\epsilon^*, \epsilon^*])$ into two sub-ranges, $(\epsilon^*, \rho)$ and $(\rho, \epsilon^*]$, and also consider the special case where $\epsilon = \rho$. In the first sub-range, $(1 + \epsilon - \rho) \mu$ is the only possible subjective valuation of the asset within $[(1 - \epsilon) \mu, \mu)$, because $\frac{p}{2} < \epsilon^* < \epsilon < \rho$ and $(1 - \rho) \mu < (1 - \epsilon) \mu$.

If $B \geq (1 + \epsilon - \rho) \mu$, the investor sells in Cases (1), (2), (3), and (5), leading to a conditional expected asset value of $(1 - \frac{(1-\lambda)p}{2\lambda + (1-\lambda)p} \epsilon) \mu$. Because $\left(1 - \frac{(1-\lambda)p}{2\lambda + (1-\lambda)p} \epsilon\right) \mu = \left(1 + \epsilon - \frac{2(p + \lambda - p\lambda)}{2\lambda + (1-\lambda)p} \epsilon\right) \mu$ and $\epsilon > \epsilon^* = \frac{2\lambda + (1-\lambda)p}{2(p + \lambda - p\lambda)} \rho$, $(1 - \frac{(1-\lambda)p}{2\lambda + (1-\lambda)p} \epsilon) \mu < (1 + \epsilon - \rho) \mu$. Therefore, any bid price weakly higher than $(1 + \epsilon - \rho) \mu$ gives a negative expected profit. If $B < (1 + \epsilon - \rho) \mu$, the investor sells in Cases (2), (3), and (5), implying a conditional expected asset value of $\left(1 - \frac{p}{p + 2(1-\rho)\lambda} \epsilon\right) \mu$. A bid price different from this conditional expected value and less than $(1 + \epsilon - \rho) \mu$ will lead to a non-zero expected profit to the market makers. Therefore, $\left(1 - \frac{p}{p + 2(1-\rho)\lambda} \epsilon\right) \mu$ is the only competitive bid price when $\epsilon^* < \epsilon < \rho$.

In the second sub-range, $(\rho, \epsilon^*)$, $(1 - \rho) \mu$ is the only possible subjective valuation within $[(1 - \epsilon) \mu, \mu)$, because $\epsilon > \rho$ and $(1 + \epsilon - \rho) \mu > \mu$. If $B \geq (1 - \rho) \mu$, the investor sells in Cases (2), (3), and (5) giving a conditional expected asset value of $\left(1 - \frac{p}{p + 2(1-\rho)\lambda} \epsilon\right) \mu$, which is weakly larger than $(1 - \rho) \mu$ because $\epsilon \leq \epsilon^* \equiv \frac{p + 2(1-\rho)\lambda}{p} \rho$. So, a bid price different from this conditional expected value
and larger than \((1 - \rho) \mu\) will lead to a non-zero profit to the market makers. If \(B < (1 - \rho) \mu\), the investor sells in Cases (3) and (5), implying a conditional expected asset value of \((1 - \epsilon) \mu\), which is less than \((1 - \rho) \mu\) because \(\epsilon > \rho\). So, a bid price different from \((1 - \epsilon) \mu\) and less than \((1 - \rho) \mu\) will lead to a non-zero expected profit to the market makers. In summary, when \(\epsilon \in (\rho, \epsilon^*)\), 
\(1 - \frac{p}{p + 2(1 - \rho) \lambda} \epsilon\) \mu and \((1 - \epsilon) \mu\) are the only two possible competitive bid prices. Since \(\epsilon \leq \epsilon^* \equiv \frac{p + 2(1 - \rho) \lambda}{p} \rho, (1 - \frac{p}{p + 2(1 - \rho) \lambda} \epsilon) \mu \geq (1 - \epsilon) \mu\). We again assume that competition leads to the higher of the two potential bid prices, \(B = (1 - \frac{p}{p + 2(1 - \rho) \lambda} \epsilon) \mu\).

When \(\epsilon = \rho\), \((1 - \epsilon) \mu = (1 - \rho) \mu\) and \((1 + \epsilon - \rho) \mu = \mu\). As discussed earlier, a competitive bid price must be within the range of \([(1 - \epsilon) \mu, \mu]\), which is equivalent to \([(1 - \rho) \mu, (1 + \epsilon - \rho) \mu]\) with \(\epsilon = \rho\). So, with a competitive bid price in this special scenario, the investor sells in Cases (2), (3), and (5), which means the conditional expected asset value is \(1 - \frac{p}{p + 2(1 - \rho) \lambda} \epsilon\) \mu. A bid price different from this conditional expected value will lead to non-zero profit to the market makers. Therefore, \(B = (1 - \frac{p}{p + 2(1 - \rho) \lambda} \epsilon) \mu\) is the unique competitive bid price when \(\epsilon = \rho\). In summary, \(B = (1 - \frac{p}{p + 2(1 - \rho) \lambda} \epsilon) \mu\) is the competitive bid price for Range (2), \(\epsilon^* < \epsilon \leq \epsilon^*\).

Range (3):

When \(\epsilon > \epsilon^* > \rho\), a bid price at \((1 - \epsilon) \mu\) leads to an investor sale in Cases (3) and (5) because \((1 - \rho) \mu > (1 - \epsilon) \mu\). This implies a conditional expected asset value of \((1 - \epsilon) \mu\), which equals the bid price. So \((1 - \epsilon) \mu\) is a possible competitive bid price.

Next, we will show that, when \(\epsilon > \epsilon^*\), \(B = (1 - \epsilon) \mu\) is the unique competitive bid price.

When \(\epsilon > \epsilon^* > \rho\), \((1 - \rho) \mu\) is the only subjective valuation of the asset within \([(1 - \epsilon) \mu, \mu]\). If \(B \geq (1 - \rho) \mu\), the investor sells in Cases (2), (3), and (5), leading to a conditional expected asset value of
\[
(1 - \frac{p}{p+2(1-p)\lambda} \varepsilon) \mu. \text{ Since } \varepsilon > \varepsilon^{**} \equiv \frac{p+2(1-p)\lambda}{p} \rho \left(1 - \frac{p}{p+2(1-p)\lambda} \varepsilon\right) \mu < (1 - \rho)\mu. \text{ Therefore, a bid price weakly larger than } (1 - \rho)\mu \text{ will lead to a negative expected profit to the market makers. If } B < (1 - \rho)\mu, \text{ the investor sells in Cases (3) and (5), implying a conditional expected asset value of } (1 - \varepsilon)\mu. \text{ So, a bid price larger than } (1 - \varepsilon)\mu \text{ and smaller than } (1 - \rho)\mu \text{ will lead to a negative expected profit to the market makers. Therefore, when } \varepsilon > \varepsilon^{**}, \text{ } (1 - \varepsilon)\mu \text{ is the unique competitive bid price.}
\]

**Proof of Proposition 1:**

When relative fundamental uncertainty is intermediate and given the competitive bid price, in equilibrium the investor does not sell if she is subject to a liquidity shock and is also informed that the asset value is high. The probability of no sale conditional on a liquidity shock is \( q = \frac{p}{2} \), and the expected cost of illiquidity is \( \frac{p}{2} \lambda \rho \mu \). As a consequence the value of the asset to the investor is reduced to \( V_1 = (1 - \frac{p}{2} \lambda \rho)\mu \). The improvement in social welfare is reduced to \( (\delta - \frac{p}{2} \lambda \rho) \mu \), and the firm will decline to complete the IPO if \( \delta < \frac{p}{2} \lambda \rho \).

When relative fundamental uncertainty is high the competitive bid price is less than the investor’s private valuation should she be subject to a liquidity shock (probability \( \lambda \)) and informed that the asset value is high (probability \( \frac{p}{2} \)). In addition, the bid price is now lower than the investor’s private valuation of \( (1 - \rho)\mu \) should she be uninformed (probability \( 1 - p \)) and subject to a liquidity shock (probability \( \lambda \)). The probability that the investor does not sell, conditional on a liquidity shock, is \( q = \frac{p}{2} + (1 - p) = \left(1 - \frac{p}{2}\right) \). The expected illiquidity costs is \( \left(1 - \frac{p}{2}\right) \lambda \rho \mu \). The value of the asset to the investor is reduced to \( V_1 = \left(1 - \left(1 - \frac{p}{2}\right) \lambda \rho\right) \mu \). The improvement in social welfare is \( (\delta - \left(1 - \frac{p}{2}\right) \lambda \rho) \mu \) and the firm will decline to complete the IPO if \( \delta < \left(1 - \frac{p}{2}\right) \lambda \rho \). Since \( \left(1 - \frac{p}{2}\right) > \frac{p}{2} \) for all \( p < 1 \), the range of the firm’s liquidity needs, \( \delta \), for which the market fails completely is larger, and the IPO price and the increase in
welfare should the IPO occur is reduced, when relative fundamental uncertainty is high rather than intermediate.

**Proof of Lemma 3:**

With the bid price set to $B^* = (1 + \epsilon - \rho)\mu$, where $\epsilon^* < \epsilon < \rho$, the investor sells at $t=1$ if she is (i) informed with good news and incurs the liquidity shock (which occurs with probability $\frac{P}{2}\lambda$), (ii) uninformed and incurs the liquidity shock (which occurs with probability $(1-p)\lambda$), or (iii) informed with bad news (which occurs with probability $\frac{P}{2}$). The probability of an investor sale is $\frac{P}{2}\lambda + (1-p)\lambda + \frac{P}{2}$, while the expected asset value conditional on a sale at is $(1 - \frac{(1-\lambda)p}{2\lambda+(1-\lambda)p}\epsilon)\mu$. The market maker’s expected monetary loss is the product of the probability of sell conditional on the contracted bid and the difference between the expectation of the asset value conditional on a sale and the contracted bid price. That is $M(B^*) = -\left(\frac{P}{2}\lambda + (1-p)\lambda + \frac{P}{2}\right) \left[\left(1 - \frac{(1-\lambda)p}{2\lambda+(1-\lambda)p}\epsilon\right)\mu - (1 + \epsilon - \rho)\mu\right] = \left(\frac{P}{2}\lambda + (1-p)\lambda + \frac{P}{2}\right) \left(\epsilon + \frac{(1-\lambda)p}{2\lambda+(1-\lambda)p}\epsilon - \rho\right) \mu = \left(\frac{P}{2}(1-\lambda) + \lambda\right) \left(\frac{(2p+\lambda-p\lambda)}{2\lambda+(1-\lambda)p}\epsilon - \rho\right) \mu = \left(\frac{P}{2}(1-\lambda) + \lambda\right) \rho \left(\frac{\epsilon}{\epsilon^*} - 1\right) \mu$, where $\epsilon^* = \frac{2\lambda+(1-\lambda)p}{2(p+\lambda-p\lambda)} \rho$. Since $\epsilon > \epsilon^*$, $M(B^*) > 0$.

When $\epsilon > \rho$, the investor sells at $t=1$ if she is (i) informed with good news and incurs a liquidity shock (which occurs with probability $\frac{P}{2}\lambda$), (ii) uninformed (which occurs with probability $(1-p)$), or (iii) informed with bad news (which occurs with probability $\frac{P}{2}$). The probability of sell is $\frac{P}{2}\lambda + (1-p) + \frac{P}{2}$, and the market maker’s expectation of the value of the asset conditional on a sale at $(1 + \epsilon - \rho)\mu$ is $(1 - \frac{(1-\lambda)p}{2(1-\lambda)p}\epsilon)\mu$. Hence, the market maker’s expected monetary loss is $M(B^*) = -\left(\frac{P}{2}\lambda + (1-p) + \frac{P}{2}\right) \left[\left(1 - \frac{(1-\lambda)p}{2(1-\lambda)p}\epsilon\right)\mu - (1 + \epsilon - \rho)\mu\right] = \left(\frac{P}{2}\lambda + (1-p) + \frac{P}{2}\right) \left[\frac{\epsilon}{\epsilon^*} + \frac{(1-\lambda)p}{2(1-\lambda)p}\epsilon - \rho\right] \mu = \left(\frac{\epsilon}{\epsilon^*} - 1\right) \mu = (\epsilon - \rho + (1 - \lambda)\frac{P}{2}\rho)\mu$. Because $\rho < \epsilon$, $M(B^*) > 0$. 

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Proof of Proposition 2:

When relative fundamental uncertainty is intermediate or high, if the firm completes the IPO and enters a DMM contract calling for a bid price $B^* = (1 + \epsilon - \rho)\mu$, the probability that the investor will not sell after a liquidity shock is reduced to $q(B^*) = 0$. With this bid price the market maker suffers an expected monetary loss as specified in Lemma 3, leading to requisite compensation to the market maker of $C = M(B^*)$. Since the market maker’s expected monetary loss is also the investor’s expected monetary gain from trading, the IPO price becomes $\mu + M(B^*)$. Therefore, the firm net gain from completing the IPO with the DMM contract is $\mu + M(B^*) - C = (1 - \delta)\mu = \delta\mu$. Without a DMM contract, the firm net gain from completing the IPO is $(1 - q(B)\lambda\rho)\mu - (1 - \delta)\mu = (\delta - q(B)\lambda\rho)\mu$, which is less than $\delta\mu$ because $q(B)\lambda\rho > 0$. Since $\delta\mu > 0$, it is optimal for the firm to complete the IPO with a DMM contract that calls for a bid price $B^* = (1 + \epsilon - \rho)\mu$, which also maximizes the social welfare gain to $\delta\mu$.

Proof of the thresholds levels of $\delta$ and $\beta$ summarized in Table 2:

Let $\pi_{com}$ denote the firm’s gain from completing the IPO and relying on competitive liquidity provision, which is $\pi_{com} = \beta\left(\delta - \lambda \frac{p}{2}\rho\right)\mu$ when $\epsilon^* < \epsilon \leq \epsilon^{**}$, and is $\pi_{com} = \beta\left(\delta - \lambda \left(1 - \frac{p}{2}\right)\rho\right)\mu$ when $\epsilon > \epsilon^{**}$. Therefore, the $\delta^*$ that yields $\pi_{com}$ equal to zero is $\lambda \frac{p}{2}\rho$ when $\epsilon^* < \epsilon \leq \epsilon^{**}$, and is $\lambda \left(1 - \frac{p}{2}\right)\rho$ when $\epsilon > \epsilon^{**}$. When $\delta < \delta^*$, $\pi_{com} < 0$. Otherwise, $\pi_{com} \geq 0$.

Let $\pi_{DMM}$ denote the firm’s gain from the IPO with a DMM contract. When $\epsilon^* < \epsilon \leq \rho$, $\pi_{DMM} = \beta\delta\mu - (1 - \beta) \ast (\epsilon(p + \lambda - p\lambda) - \rho \left(\frac{1}{2}p + \lambda - \frac{1}{2}p\lambda\right))\mu$. If $\delta < \delta^* (\pi_{com} < 0)$, the threshold level of $\beta$, which makes the firm indifferent between completing the IPO with a DMM contract and forgoing the IPO, should satisfy $\pi_{DMM} = 0$. Therefore, in this case, the threshold $\beta^*$ is

$$
\frac{\epsilon(p + \lambda - p\lambda) - \rho \left(\frac{1}{2}p + \lambda - \frac{1}{2}p\lambda\right)}{\delta + \epsilon(p + \lambda - p\lambda) - \rho \left(\frac{1}{2}p + \lambda - \frac{1}{2}p\lambda\right)}
$$

If $\delta \geq \delta^*(\pi_{com} \geq 0)$, the threshold level of $\beta$, which makes the firm
indifferent between completing the IPO with and without a DMM contract, should satisfy $\pi_{DMM} = \pi_{com}$.

Therefore, in this case, the threshold $\beta^*$ is

$$
\frac{\epsilon(p+\lambda-p\lambda)-\rho\left(\frac{1}{2}p+\lambda-\frac{1}{2}p\lambda\right)}{\lambda\rho+\epsilon(p+\lambda-p\lambda)-\rho\left(\frac{1}{2}p+\lambda-\frac{1}{2}p\lambda\right)}.
$$

Similarly, when $\rho < \epsilon < \epsilon^*$ or $\epsilon > \epsilon^*$, $\pi_{DMM} = \beta\delta\mu - (1 - \beta)\left(\epsilon - \rho + (1 - \lambda)\frac{P}{2}\rho\right)\mu$. If $\delta < \delta^*(\pi_{com} < 0)$, the threshold level of $\beta$, which makes the firm indifferent between completing the IPO with a DMM contract and forgoing the IPO, should satisfy $\pi_{DMM} = 0$. Therefore, in this case, the threshold $\beta^*$ is

$$
\frac{\epsilon - \rho + (1 - \lambda)\frac{P}{2}\rho}{\delta + \epsilon - \rho + (1 - \lambda)\frac{P}{2}\rho}.
$$

If $\delta \geq \delta^*(\pi_{com} \geq 0)$, the threshold level of $\beta$, which makes the firm indifferent between completing the IPO with and without a DMM contract, should satisfy $\pi_{DMM} = \pi_{com}$.

When $\rho < \epsilon \leq \epsilon^*$, $\pi_{com} = \beta\left(\delta - \lambda\frac{P}{2}\rho\right)\mu$. Therefore the threshold $\beta^* = \frac{\epsilon - \rho + (1 - \lambda)\frac{P}{2}\rho}{\epsilon - \rho + (1 - \lambda)\frac{P}{2}\rho}$. When $\epsilon > \epsilon^*$, $\pi_{com} = \beta\left(\delta - \lambda\left(1 - \frac{P}{2}\rho\right)\right)\mu$. Therefore, the threshold $\beta^* = \frac{\epsilon - \rho + (1 - \lambda)\frac{P}{2}\rho}{\epsilon - \rho + (1 - \lambda)\frac{P}{2}\rho + \lambda\frac{P}{2}\rho + \lambda(1 - p)\rho}$.

The derivatives of $\beta^*$ with respect to the exogenous parameters $\lambda$, $\rho$, and $p$:

When $\epsilon^* < \epsilon \leq \rho$ and $\delta < \lambda\frac{P}{2}\rho$, $\beta^* = \frac{\epsilon(p+\lambda-p\lambda)-\rho\left(\frac{1}{2}p+\lambda-\frac{1}{2}p\lambda\right)}{\lambda\rho+\epsilon(p+\lambda-p\lambda)-\rho\left(\frac{1}{2}p+\lambda-\frac{1}{2}p\lambda\right)}$. We have

$$
\frac{d\beta^*}{dp} = \frac{2\delta(2\epsilon - \rho)(1 - \lambda)}{(2\delta + 2\epsilon\lambda - 2\epsilon\lambda p - 2\rho\lambda + \lambda\rho)^2} \geq 0 \text{ because } \frac{p}{2} < \epsilon^* < \epsilon \text{ and } 0 < \lambda \leq 1; \quad 23
$$

$$
\frac{d\beta^*}{d\lambda} = \frac{2\delta(2\epsilon - \rho)(1 - \rho - pp)}{(2\delta + 2\epsilon\lambda - 2\epsilon\lambda p - 2\rho\lambda + \lambda\rho)^2} < 0 \text{ because } \epsilon \leq \rho \text{ and } 0 < p \leq 1; \quad \text{ and}
$$

$$
\frac{d\beta^*}{d\rho} = \frac{2\delta(\lambda-p-2\lambda-p)}{(2\delta + 2\epsilon\lambda - 2\epsilon\lambda p - 2\rho\lambda + \lambda\rho)^2} < 0 \text{ because } 0 < p \leq 1 \text{ and } 0 < \lambda \leq 1.
$$

When $\epsilon^* < \epsilon \leq \rho$ and $\delta \geq \lambda\frac{P}{2}\rho$, $\beta^* = \frac{\epsilon(p+\lambda-p\lambda)-\rho\left(\frac{1}{2}p+\lambda-\frac{1}{2}p\lambda\right)}{\lambda\rho+\epsilon(p+\lambda-p\lambda)-\rho\left(\frac{1}{2}p+\lambda-\frac{1}{2}p\lambda\right)}$. We have

$$
\frac{d\beta^*}{dp} = \frac{2\rho\lambda^2(p-\epsilon)}{(2\epsilon\lambda - 2\epsilon\lambda p - 2\rho\lambda + 2\lambda p)^2} \geq 0 \text{ because } \epsilon \leq \rho; \quad 24
$$

23 The equality arises when $\lambda = 1$. Here, the investor always incurs a liquidity shock and always sells if the bid price is set to $B^* = (1 + \epsilon - \rho)\mu$. In this case the probability of receiving private information no longer affects the equilibrium, and $\frac{d\beta^*}{dp} = 0$.  

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\[
\frac{d\beta^*}{d\lambda} = \frac{\rho p(\rho - 2\varepsilon)}{(2\varepsilon p + 2\lambda - 2\varepsilon \lambda p - \rho - 2\varepsilon p + 2\lambda p)^2} < 0 \text{ because } \frac{\rho}{2} < \varepsilon^* < \varepsilon; \text{ and}
\]
\[
\frac{d\beta^*}{dp} = \frac{2\varepsilon \lambda p(\lambda p - 2\varepsilon p - \rho - 2\varepsilon p + 2\lambda p)^2} < 0 \text{ because } 0 < p \leq 1, 0 < \varepsilon \leq 1, \text{ and } 0 < \lambda \leq 1.
\]

When \( \rho < \varepsilon \leq \varepsilon^* \) and \( \delta < \lambda \frac{\rho}{2} \), and when \( \varepsilon > \varepsilon^* \) and \( \delta < \lambda \left(1 - \frac{\rho}{2}\right) \), \( \beta^* = \frac{e - \rho + (1 - \lambda)\frac{\rho}{2p}}{\delta + e - \rho + (1 - \lambda)\frac{\rho}{2p}} \). We have
\[
\frac{d\beta^*}{dp} = \frac{2\rho \delta (1 - \lambda)}{(2\delta + 2\varepsilon - 2\rho + \rho p - \lambda p)^2} \geq 0 \text{ because } 0 < \lambda \leq 1 \text{ and } 0 < p \leq 1; \]
\[
\frac{d\beta^*}{d\lambda} = \frac{-2p \rho \delta}{(2\delta + 2\varepsilon - 2\rho + \rho p - \lambda p)^2} < 0; \text{ and}
\]
\[
\frac{d\beta^*}{dp} = \frac{-2(2\rho - \rho p) \delta}{(2\delta + 2\varepsilon - 2\rho + \rho p - \lambda p)^2} < 0 \text{ because } 0 < \lambda \leq 1 \text{ and } 0 < p \leq 1.
\]

When \( \rho < \varepsilon \leq \varepsilon^* \) and \( \delta \geq \lambda \frac{\rho}{2} \), \( \beta^* = \frac{e - \rho + (1 - \lambda)\frac{\rho}{2p}}{\lambda p + e - \rho + (1 - \lambda)\frac{\rho}{2p}} \). We have
\[
\frac{d\beta^*}{dp} = \frac{2\rho \lambda (\rho - e)}{(2\varepsilon - 2p + \rho p)^2} < 0 \text{ because } \rho < \varepsilon; \]
\[
\frac{d\beta^*}{d\lambda} = \frac{-pp}{2\varepsilon - 2p + \rho p} < 0 \text{ because } \rho < \varepsilon; \text{ and}
\]
\[
\frac{d\beta^*}{dp} = \frac{-2 \lambda p}{(2\varepsilon - 2p + \rho p)^2} < 0.
\]

When \( \varepsilon > \varepsilon^* \) and \( \delta \geq \lambda \left(1 - \frac{\rho}{2}\right) \), \( \beta^* = \frac{e - \rho + (1 - \lambda)\frac{\rho}{2p}}{e - \rho + (1 - \lambda)\frac{\rho}{2p} + \lambda \frac{\rho}{2p} + \lambda (1 - p) \rho} \). We have
\[
\frac{d\beta^*}{dp} = \frac{2\lambda \rho (\rho - \lambda p)}{(2\varepsilon - 2p + \rho p - 2\lambda p + 2\lambda p)^2} > 0 \text{ because } \varepsilon > \rho; \]
\[
\frac{d\beta^*}{d\lambda} = \frac{\rho (p - 2) (\rho p - 2p + 2\varepsilon)}{(2\varepsilon - 2p + \rho p - 2\lambda p + 2\lambda p)^2} < 0 \text{ because } \varepsilon > \rho; \text{ and}
\]
\[
\frac{d\beta^*}{dp} = \frac{2 \lambda \rho (p - 2)}{(2\varepsilon - 2p + \rho p - 2\lambda p + 2\lambda p)^2} < 0.
\]

24 The equality arises when \( \rho = \varepsilon \). The intuition is the following. With \( \delta \geq \lambda \frac{\rho}{2} \), the net gain from completing the IPO with competitive market makers is positive, so the decision whether to engage a market maker depends on the comparison between net gains from completing the IPO with and without a DMM contract. The net gain from completing the IPO with and without a DMM contract decrease with \( p \) at the same speed when \( \rho = \varepsilon \), so the difference between the net gains of these two options does not change with \( p \). This leads to a derivative of \( \beta^* \) with respect to \( p \) equal to zero.
A possible DMM contract calling for a bid quote $\hat{B} = (1 - \rho)\mu < B^*$. 

In this model, aggregate welfare gains are altered at certain levels of $B$, but are constant within these breakpoints. We show below that there are only two potentially relevant bid prices that are associated with shifts in aggregate welfare. These are $B^* = (1 + \epsilon - \rho)\mu$ and $\hat{B} = (1 - \rho)$. We focus for simplicity on the smallest bid price that generates a given possible welfare gain. In general an increase in the bid price leads to a higher IPO price, greater market maker losses, and a larger requisite payment from the firm to the market maker, but these increased cash flows are zero-sum except at the discrete points identified.

When relative fundamental uncertainty is intermediate, $B^* = (1 + \epsilon - \rho)\mu$ is the only breakpoint relevant to the analysis, and the firm chooses between no IPO, IPO without a DMM contract, or IPO with a DMM contract calling for a bid quote equal to $B^*$. When relative fundamental uncertainty is high the competitive bid quote is less than $\hat{B} = (1 - \rho)\mu$, giving rise to a fourth possibility, that the firm completes the IPO but contracts for a bid quote of $\hat{B} < B^*$. When $\beta = 1$ the firm would not consider this alternative, but would maximize their gain from trade by contracting for $B = B^*$. However, when $\beta < 1$ the firm might prefer to contract for $B = \hat{B}$. In this case, $q(\hat{B}) > 0$ and $C = M(\hat{B}) > 0$, and the firm’s net gain if it conducts the IPO and enters the DMM agreement for this lower bid price $\hat{B}$ is $\pi = \beta\mu(\delta - q(\hat{B})\lambda\rho) - M(\hat{B})(1 - \beta)$. Recall that the firm’s net gain if it conducts the IPO without a DMM agreement is $\pi = \beta\mu(\delta - q(B)\lambda\rho)$ and the firm’s net gain if it conducts the IPO with a DMM agreement for $B^*$ is $\pi = \beta\mu(\delta - q(B^*)\lambda\rho) - M(B^*)(1 - \beta)$.

Complete market failure occurs if the firm’s net gain from completing the IPO is negative without or with a DMM agreement calling for a bid price at either $B^*$ or $\hat{B}$. That is, the market fails completely if $\delta < q(B)\lambda\rho$ and $\beta < \min\left\{\frac{M(B^*)}{M(B^*) + \mu\delta}, \frac{M(\hat{B})}{M(\hat{B}) + \mu\delta - q(\hat{B})\lambda\rho\mu}\right\}$.
Partial market failure, in the form that the firm completes the IPO but does not enter any DMM agreement, occurs if the firm’s net gain without a DMM agreement is positive and higher than the net gain with a DMM agreement calling for a bid price at either \( B^* \) or \( \hat{B} \). That is \( \delta \geq q(B)\lambda \rho \) and

\[
\beta < \min \left\{ \frac{M(B^*)}{M(B^*) + q(B)\lambda \mu}, \frac{M(\hat{B})}{M(\hat{B}) + (q(B) - q(\hat{B}))\lambda \mu} \right\}.
\]

Partial market failure, in the form that the firm completes the IPO and enters a DMM agreement calling for \( \hat{B} = (1 - \rho)\mu \) rather than \( B^* \), occurs if the firm’s net gain with a DMM agreement calling for \( \hat{B} \) is positive and higher than the net gain without a DMM agreement or with a DMM agreement calling for \( B^* \). This occurs if \( \max \left\{ \frac{M(\hat{B})}{M(\hat{B}) + (q(B) - q(\hat{B}))\lambda \mu}, \frac{M(B^*)}{M(B^*) + (q(\hat{B}) - q(B))\lambda \mu} \right\} < \beta < \frac{M(B^*) - M(\hat{B})}{M(B^*) - M(\hat{B}) + q(\hat{B})\lambda \mu} \).

Stated alternatively, the firm would choose to complete the IPO with a DMM agreement calling for a bid price at \( B^* = (1 + \epsilon - \rho)\mu \) if

\[
\beta \geq \max \left\{ \frac{M(B^*)}{M(B^*) + \mu \delta}, \frac{M(B^*)}{M(B^*) + q(B)\lambda \mu}, \frac{M(B^*) - M(\hat{B})}{M(B^*) - M(\hat{B}) + q(\hat{B})\lambda \mu} \right\}.
\]

Thus, consistent with the simpler case addressed in the main text, the market fails partially or completely if the firm’s bargaining power is less than a critical level that depends on the magnitude of the required payment to the market maker.

**Proof that the only two bid prices that are associated with shifts in aggregate welfare are \( B^* \) and \( \hat{B} \).**

In principle the firm could contract with the DMM to quote at any price greater than the competitive quote. We confine our attention to the economically relevant cases which range from the competitive bid quote to the highest possible asset value \((1 + \epsilon)\mu\).

When relative fundamental uncertainty is high \( \epsilon > \epsilon^{**} = \frac{p + 2(1-p)\lambda}{p}\rho \), the competitive bid price is \((1 - \epsilon)\mu\). Because \((1 - \epsilon)\mu < (1 - \rho)\mu < (1 + \epsilon - \rho)\mu\), the investor sells in cases (2) and (5) displayed in Figure 1. The range of possible contract prices is \((1 - \epsilon)\mu < B \leq (1 + \epsilon)\mu\). When \( B \in \)
[((1 - \epsilon)\mu, (1 - \rho)\mu), the investor sells in case (2) and (5). This trading behavior is the same as that with competitive bid price \((1 - \epsilon)\mu\), implying that a contracted bid price in this range does not improve liquidity or welfare.

When \(B \in [(1 - \rho)\mu, \mu]\), the investor sells in cases (2), (3), and (5). Because the investor changes from no sell to sell in case (3) at a contracted price \(B \in [(1 - \rho)\mu, \mu]\), the illiquidity cost is reduced from \((1 - \frac{p}{2})\lambda \rho \mu\) to \(\frac{p}{2} \lambda \rho \mu\) at that point. Any contracted bid price within this range will lead to the same reduction, \((1 - p)\lambda \rho \mu\), in expected illiquidity cost. When \(\beta = 1\) the firm’s net gain is \((1 - p)\lambda \rho \mu\) and the firm is indifferent among contracted bid prices in the region \([(1 - \rho)\mu, \mu]\). However, when \(\beta < 1\), the compensation to the market maker is not fully recovered from the higher IPO price. The net gain from the IPO is \(\beta(1 - p)\lambda \rho \mu - (1 - \beta)M(B)\), which decreases with \(B\). Therefore, the firm prefers \(B = (1 - \rho)\mu\) to other contract price in the region \([(1 - \rho)\mu, \mu]\).

When \(B \in [\mu, (1 + \epsilon - \rho)\mu]\), the investor sells in cases (2), (3), (4), and (5). Because the investor changes from no sell to sell in case (3) with a contracted price \(B \in [\mu, (1 + \epsilon - \rho)\mu]\), the illiquidity cost is reduced from \((1 - \frac{p}{2})\lambda \rho \mu\) to \(\frac{p}{2} \lambda \rho \mu\). Note that the change from no trade to trade in case (4) does not change the expected illiquidity cost because there is no liquidity shock in case (4). With a bid price in the range of \([\mu, (1 + \epsilon - \rho)\mu]\), the reduction in illiquidity cost is the same as that from a bid price \(B = (1 - \rho)\mu\), while the required compensation is higher. Therefore, the firm prefers \(B = (1 - \rho)\mu\) to any contract price in the region \([\mu, (1 + \epsilon - \rho)\mu]\).

When \(B \in [(1 + \epsilon - \rho)\mu, (1 + \epsilon)\mu]\), the investor sells in cases (1), (2), (3), (5), and (6). Because the investor always sells after incurring a liquidity shock with a contracted price \(B \in [(1 + \epsilon - \rho)\mu, (1 + \epsilon)\mu]\), the illiquidity cost is reduced from \((1 - \frac{p}{2})\lambda \rho \mu\) to zero. Given \(\beta = 1\), the firm is indifferent among any price in the region \([(1 + \epsilon - \rho)\mu, (1 + \epsilon)\mu]\), while the firm prefers \(B = (1 + \epsilon - \rho)\mu\) when \(\beta < 1\). Therefore, when relative fundamental uncertainty is high there are only two relevant contract prices, \(B^* = (1 + \epsilon - \rho)\mu\) and \(\bar{B} = (1 - \rho)\).

When relative fundamental uncertainty is intermediate, the competitive bid price is \(\left(1 - \frac{p}{p + 2(1 - \rho)\lambda}\epsilon\right)\mu\), which is weakly larger than \((1 - \rho)\mu\). Therefore, \((1 - \rho)\mu\) is not a liquidity improving bid price. As discussed in the case with high relative fundamental uncertainty, \(B = (1 + \epsilon - \rho)\mu\) weakly dominates other possible contract prices. Therefore we need only consider one possible contract price \(B^* = (1 + \epsilon - \rho)\mu\) when relative fundamental uncertainty is intermediate.