# Risk and Return in Equity and Options Markets

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**Job Market Paper**

**Abstract**

I examine the role of a market-wide volatility factor in the pricing of the cross-section of returns on individual stock options. While it is commonly accepted that option prices depend upon the volatility of the underlying asset, recent evidence in the literature suggests that it is not clear whether market-wide volatility is a priced factor in individual options. While some studies have found a volatility risk premium implicit in index option prices, efforts to document the same type of premium using individual stock options have uncovered little supporting evidence. Applying an improved test design, I show that market-wide volatility is an economically and statistically important priced risk factor in the cross-section of stock option returns. This evidence supports recent theories of market-wide volatility as a state factor.

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# 1 Introduction

The role of volatility risk in markets has been intensely studied in the recent literature. Evidence from the cross-section of equity returns suggests a negative price of risk for market-wide volatility, meaning that investors are willing to accept lower expected returns on stocks that hedge increases in market volatility.[1] Evidence from index options also suggests a negative price of volatility risk.[2] Surprisingly however, the volatility risk premium implicit in individual stock options does not appear to coincide with the premium implied by index options.[3] Attempts to cross-sectionally identify a negative price of market-wide volatility risk using stock options have also met with little success.[4] Taken together these results are puzzling, especially when such a tight relationship exists between options and their underlying stocks.

The options market offers an ideal setting in which to study the pricing impact of systematic volatility. While far less studied than index options, individual options offer a much richer cross-section with which to study variation in returns because they vary at the firm level in addition to the contract level. Furthermore, option prices depend critically on volatility. Together these facts suggest that using individual stock options data improves the potential of accurately estimating the price of market-wide volatility risk in the cross-section.

While stock options offer a very promising asset class with which to study the price of market-wide volatility and potentially other market-wide risks, relatively little is known about the systematic factors that determine their expected returns. In fact, several papers

[1]Ang, Hodrick, Xing, and Zhang (2006), Adrian and Rosenberg (2008), Drechsler and Yaron (2011), Dittmar and Lundblad (2014), Boguth and Kuehn (2013), Campbell, Giglio, Polk, and Turley (2012) and Bansal, Kiku, Shaliastovich, and Yaron (2013) study the role of market-wide volatility risk in the cross-section of equity returns.

[2]See Bakshi and Kapadia (2003a) and Coval and Shumway (2001).

[3]See Bakshi and Kapadia (2003b), Carr and Wu (2009) and Driessen, Maenhout, and Vilkov (2009).

[4]Using delta-hedged individual option returns, Duarte and Jones (2007) find no significant price of volatility risk *orthogonal* to underlying assets in unconditional models but a significant price in conditional models. Da and Schaumburg (2009) and Di Pietro and Vainberg (2006) estimate the price of volatility risk in the cross-section of option-implied variance swap returns but find opposite signs for the price of risk. Driessen, Maenhout, and Vilkov (2009) argue that returns on individual options are largely orthogonal to the part of market-wide volatility that is priced in the cross-section.

have offered strong evidence that options are not redundant securities.[5] Coupled with evidence that option returns exhibit some surprising patterns[6] as well as demand-based option pricing,[7] this suggests that returns on options are not determined in exactly the same way as returns on their underlying stocks. Thus, it is important to independently show that market volatility is priced in the cross-section of returns of stock options. If it is not priced in the cross-section of a large class of assets like stock options (as has been suggested in the literature) it would be difficult to make a compelling argument that market-wide volatility is a state factor.

I empirically investigate the price of market-wide volatility risk in both the equity and options markets. Specifically, I empirically address two questions: 1) Is a market-wide volatility factor priced in the cross-section of equity option returns? 2) Is the price of volatility risk the same in the equity and option markets? It is important to distinguish between the systematic risk associated with market-wide volatility and the stock-specific measure of asset volatility, which is often included in models of option prices. I study whether investors are willing to pay a premium for individual stock options that hedge market volatility whereas it is commonly accepted that investors are willing to pay a premium for options whose underlying stocks are volatile. My results show that even though the volatility risk premium extracted from individual stock options data does not appear to be consistent with that of index options, systematic volatility is priced in the cross-section of stock option returns. This supports the notion of volatility as a state factor.

To answer the questions stated above, I first create a new set of option portfolios that are optimally designed to facilitate econometric inference and to identify the price of market volatility. Following Constantinides, Jackwerth, and Savov (2013), I adjust the realized returns of each option in order to reduce the effect of contract-level leverage. This paper is the first to apply this leverage adjustment to individual option returns instead of index option returns. The leverage adjustment is econometrically important because it reduces the effect

[5] See for example Bakshi, Cao, and Chen (2000), Buraschi and Jackwerth (2001) and Vanden (2004).
[6] See Ni (2008) and Boyer and Vorkink (2014)
[7] See Garleanu, Pedersen, and Poteshman (2009) and Bollen and Whaley (2004).

of outliers that arise due to the extreme leverage especially inherent in out-of-the-money options. Furthermore, the adjustment helps to stabilize the stochastic relation between option returns and time-varying risk factors. I also propose a new method of sorting options that results in highly dispersed sensitivity of portfolio returns to market-wide volatility. The combination of forming portfolios of options and leverage-adjusting each option's returns renders standard econometric techniques feasible. This allows me to examine option returns in a manner typical of cross-sectional studies of stock returns as opposed to the highly stylized and non-linear models typically used in the option pricing literature.

Using GMM, I test a wide range of stochastic discount factors (SDFs) while controlling for factors commonly used to explain the cross-section of stock returns.[8] In addition to augmenting classical linear models with a volatility factor, I also posit SDFs that include factors from the literature that capture tail risk in equity returns. These factors help to disentangle volatility risk from the risk of market downturns, controlling for the well-documented leverage effect whereby market-wide volatility increases when market returns are negative. I show that market-wide volatility is an extremely important and robust risk factor in the cross-section. I then compare estimated prices of risk between the equity and options markets.

My results regarding a priced volatility factor align with the argument that market-wide volatility is a state factor. However, I find evidence that the price of volatility risk in the options market is larger in magnitude than in the stock market. This is somewhat surprising given that others have found volatility risk in options to be non-distinguishable from zero or to even take the opposite sign. My results are consistent with the demand-based option pricing theory of Bollen and Whaley (2004) and Garleanu, Pedersen, and Poteshman (2009) whereby intermediaries facing high demand for options charge larger premiums in order

[8] I use the Generalized Method of Moments (GMM) in cross-sectional tests because it has several advantages over alternative asset pricing tests when studying option returns. For example, options of different moneyness tend to exhibit different levels of volatility. Thus standard errors from OLS cross-sectional regression cannot be applied to options due to heteroskedasticity of test assets. Furthermore, because the sensitivity of an option to time-varying risk factors can dramatically vary with option-specific parameters, time series regressions used in the first stage of Fama and MacBeth (1973) regressions may be very unreliable when using options data. GMM does not rely on a first stage time-series to explicitly estimate betas. In fact applying GMM only requires stationary and ergodic test assets.

to cover positions that cannot be perfectly hedged. As stochastic volatility is a possibly unhedgeable risk that dealers face, my findings may be the result of equilibrium pricing in the market due to market incompleteness. An alternative explanation is simply that there are limits to arbitrage preventing this apparent mispricing from being arbitraged away. This explanation is consistent with Figlewski (1989) who shows that arbitrage opportunities in option markets are costly and often too expensive to exploit in practice. A third explanation is that the two markets are segmented in such a way that market participants who are willing to pay more to hedge volatility invest in options.

The remainder of the paper is organized as follows. Section 2 describes the data used in the paper and the construction of factors used in the econometric analysis. Section 3 describes the test assets used throughout the paper. Sections 4 and 5 present the results. Section 6 concludes.

# 2 Data

This section describes the data used in the study. I begin by describing the data sources. I then describe the filters used to clean the raw data. Finally, I describe the formation and properties of risk factors used throughout the paper.

## 2.1 Data Sources

Options data for the paper are from the OptionMetrics Ivy DB database. I use equity options for the analysis of the cross-section of option returns. I also use index options on the S&P 500 to construct factors used in the analysis. The OptionMetrics database begins in January 1996 and currently runs through August 2013. Data include daily closing bid and ask quotes, open interest, implied volatility and option greeks. The greeks and implied volatility for European style options on the S&P 500 are computed by OptionMetrics using the standard Black-Scholes-Merton model, while implied volatilities and greeks for individual options are computed using the Cox, Ross, and Rubinstein (1979) binomial tree method. The

OptionMetrics security file contains data on the assets underlying each option in the data. These data include closing prices, daily returns and shares outstanding for each underlying stock. For the construction of stock portfolios, I use the entire universe of CRSP stocks over the same time period as the OptionMetrics data.

As is typical in the empirical options literature, I use options data only for S&P 500 firms. This partially eliminates the problem of illiquidity in options data. I follow the convention in the literature and calculate option price estimates by taking the midpoint between closing bid and ask quotes each day. Since the dates I use for monthly holding period returns are not the first and last trading day of a calendar month, I use the daily factor and portfolio data from Kenneth French's website to construct monthly holding period returns for factors and portfolios alike. The risk-free rate I use throughout the paper is also taken from Kenneth French's website.

## 2.2 Data Filters

Option deltas ($\Delta$) measure the sensitivity of on option's price to small movements in the underlying stock. Formally, this is equivalent to defining the delta of an option as the partial derivative of the option price with respect to the price of the underlying stock. For a given underlying stock, the delta of put or call options is a monotonic function of option moneyness. With this logic in mind, I follow the convention in the literature and define option moneyness according to the option's delta as reported by OptionMetrics. Out of the money (OTM), at the money (ATM) and in the money (ITM) puts and calls are defined throughout the paper by the following:

$$\begin{array}{rlrl} \text{OTM calls:} & 0.125 < \Delta \leq 0.375 & \text{OTM puts:} & -0.375 < \Delta \leq -0.125 \\ \text{ATM calls:} & 0.375 < \Delta \leq 0.625 & \text{ATM puts:} & -0.625 < \Delta \leq -0.375 \\ \text{ITM calls:} & 0.625 < \Delta \leq 0.875 & \text{ITM puts:} & -0.875 < \Delta \leq -0.625. \end{array}$$

I follow Goyal and Saretto (2009), Christoffersen, Goyenko, Jacobs, and Karoui (2011) and Cao and Han (2013) among others in my data filtering procedure. First I eliminate options for which the bid price is greater than the ask price or where the bid price is equal to zero. Next I remove all observations for which the bid ask spread is below the minimum tick size. The minimum tick size is \$0.05 for options with bid ask midpoint below \$3.00 and is \$0.10 for options with bid ask midpoint greater than or equal to \$3.00. In order to further reduce the impact of illiquid options, I remove all options with zero open interest. I also remove any options for which the implied volatility or option delta is missing.

Finally, in order to reduce the impact of options that are exercised early, I follow Frazzini and Pedersen (2012) by eliminating options that are not likely to be held to maturity. This is done by first calculating each option's intrinsic value $V = (S-K)^+$ for calls and $V = (K-S)^+$ for puts, where $K$ is the option's strike price and $S$ is the price of the underlying stock. I then eliminate all options for which the time value, defined by $\frac{(P-V)}{P}$, is less than 0.05 one month before expiration, where $P$ denotes the price of the option (estimated by the bid-ask midpoint). Table 1 gives summary statistics for the filtered options data.

## 2.3 Option Returns Calculation

Equity options expire on the Saturday following the third Friday of each month. I compute option returns over a holding period beginning the first Monday following an expiration Saturday and ending the third Friday of the following month. Even though all options in the sample are American and therefore have the option to exercise early, I follow the majority of the literature on option returns and assume all options are held until expiration. The removal of options with low "time value" described above and in Frazzini and Pedersen (2012) attempts to remove those options that are likely to be exercised early and not held until the following month's expiration date.

The payoff to the option is calculated using the cumulative adjustment factor to adjust for any stock splits that occur over the holding period. Put and call options' gross returns over the month $t$ are given by

$$R^C_{t+\tau} = max\left\{0, S_{t+\tau}\left(\frac{CAF_{t+\tau}}{CAF_t}\right) - K\right\}\Big/ P_t, \tag{1}$$

and

$$R^P_{t+\tau} = max\left\{0, K - S_{t+\tau}\left(\frac{CAF_{t+\tau}}{CAF_t}\right)\right\} \Big/ P_t, \tag{2}$$

where $\tau$ is the time to maturity.

### 2.4 Factor Construction

Following Ang, Hodrick, Xing, and Zhang (2006) and Chang, Christoffersen, and Jacobs (2013), I base my measure of market-wide volatility on the VIX index. Since the VIX exhibits a high level of autocorrelation, innovations in the VIX can simply be estimated by first differences, $\Delta VIX_t = VIX_t - VIX_{t-1}$. Throughout the paper I use VIX/100 because the VIX is quoted in percentages. This way I use a measure of market volatility as opposed to market volatility scaled by 100. Innovations in the VIX are highly negatively correlated with the market factor. This is the well known "Leverage Effect." In order to ensure that the volatility factor I use is not simply picking up negative movements in the market level, I further follow Chang, Christoffersen, and Jacobs (2013) by orthogonalizing innovations in the VIX with respect to market excess returns. This is simply done by regressing $\Delta VIX$ on market excess returns and taking the residuals as the orthogonalized volatility factor. This orthogonalized measure of innovations in the VIX is the volatility factor referred to throughout the paper.

I construct market-wide jump and volatility-jump factors following Constantinides, Jackwerth, and Savov (2013). The jump factor is defined as the sum of all daily returns on the S&P 500 that are below $-4\%$ in a given month. Since each month in my sample begins immediately following an option expiration date and ends at the following option expiration date, the jump factor is simply the sum of all daily returns in this time span for which returns are below the $-4\%$ threshold. If no such days exist, then the jump factor is zero for the month. Approximately 7% of the months in the sample have a non-zero jump factor. Finally, I include a volatility jump factor which captures large upward jumps in volatility of the market. To construct the volatility jump factor, I take all ATM call options on the S&P 500 and calculate the equally weighted average of implied volatilities over all options between 15 and 45 days to maturity. This gives me a series of daily average implied volatilities of ATM call options. Over each holding period I then take the sum of daily changes in implied volatility for all days in which the change is greater than 0.04. Approximately 29% of months in the sample have non-zero volatility jump.

Downside risk has been proposed as a state variable in the ICAPM and has been shown to perform very well for pricing stocks in Ang, Chen, and Xing (2006) and across a number of additional asset classes including currencies, bonds and commodities in Lettau, Maggiori, and Weber (2013). I follow Lettau, Maggiori, and Weber (2013) by defining a down state to be any month in which market returns are below the mean of monthly returns over the sample period by an amount exceeding one standard deviation of returns over the sample period. The down-state factor is simply equal to returns on the CRSP value-weighted index in periods when the returns are below the down state threshold. In all other months the factor is zero. This gives a factor that is very similar to the jump factor. The main difference between the two is that the jump factor is computed using daily data to determine when the market has experienced a jump. The magnitude of negative daily returns required to be considered a jump is much more extreme than the one standard deviation measure used to establish a down state. Furthermore, because jumps are defined at a daily frequency, they can more convincingly be considered jumps in the return process as opposed to simply months where the market slowly declines. Approximately 13% of months in the sample have non-zero down-side risk.

Finally I include model-free, implied risk-neutral skewness as a down-side risk. I follow Bakshi, Kapadia, and Madan (2003) to construct a measure of risk-neutral market-wide skewness. I then take innovations of the skewness factor by estimating an ARMA(1,1) model and taking residuals of the estimates. I use these residuals as an additional control for the main tests of volatility risk.

Figure 6 shows the time series of each of the volatility, jump, volatility-jump, down-side and skew factors. Panel B shows the orthogonalized volatility factor with the original, non-orthogonal factor in the background. Each of the factors has its largest spike during the recent financial crisis. More recently there are fairly large spikes during the U.S. debt-ceiling crisis in August of 2011. Volatility and volatility-jump experienced very large jumps around the terrorist attack of September 11, 2001. Table 5 gives pairwise correlations of the three factors as well as the Fama-French and Momentum factors. The construction of the latter factors are described in the appendix.

# 3 Portfolio Construction and Summary Statistics

In order to study the determinants and behavior of risk premia in the cross-section of option returns I construct 36 portfolios of options that are sorted along three dimensions. The portfolios are constructed in order to give dispersion in mean returns and exposure to changes in the VIX.

## 3.1 Portfolio Construction

I form portfolios of options by first dividing the options into six bins according to type: calls and puts, and three moneyness categories as defined in Section 2.2. Within each of these six bins I sort into another six portfolios according to each contract's Black-Scholes-Merton implied volatility premium. For each option $k$ on stock $j$, I measure the implied volatility premium (IVP) by

$$IVP_{j,k} = \sigma_{j,k}^{BSM} - \sigma_j^{Hist},$$

where $\sigma_{j,k}^{BSM}$ denotes the Black-Scholes-Merton implied volatility extracted from option $k$'s price and $\sigma_j^{Hist}$ is the historical volatility of the underlying stock. I estimate $\sigma_j^{Hist}$ from daily returns over the previous year leading up to the beginning of each holding period.

The IVP measure is similar to the sorting measure of Goyal and Saretto (2009) but rather than measuring the ratio of implied volatility to historical volatility of the underlying, I take the difference, which represents the premium due to model-implied volatility in excess of historical volatility. Another difference between the way I sort options and the method employed by Goyal and Saretto (2009) is that I sort at the contract level as opposed to just taking a single at-the-money option for each underlying stock and comparing the two. This gives my set of portfolios greater dispersion in loadings on innovations in the VIX than does the set of portfolios studied in Goyal and Saretto (2009).

To construct a set of equity portfolios, I follow Ang, Hodrick, Xing, and Zhang (2006). I use the entire universe of CRSP stocks to double sort stocks according to their loadings on the market excess return and changes in the VIX. On the first day of each holding period I calculate the CAPM betas of each firm over the previous month's daily returns. I only include firms for which CRSP reports returns on every trading day over the previous month. The stocks are divided into two bins according to their loading on the market factor. Within each bin I then estimate a two factor model with market excess returns and changes in the VIX over the previous month and sort into six portfolios based on loadings on the second factor within each market loading category. This gives a total of twelve portfolios. I choose twelve portfolios so that they can be compared with the twelve ATM option portfolios. I choose to divide first into two market loading bins and then into six VIX innovation portfolios in order to maximize dispersion in loadings on volatility innovations while still double sorting in the manner of Ang, Hodrick, Xing, and Zhang (2006). Once the portfolios are formed, they are held for the one month holding period for which value-weighted returns are calculated. At the end of the month, the portfolios are rebalanced.

In unreported results, I find that sorting according to the systematic risk preimium described in Duan and Wei (2009) produces similar results to those described in Section 4. Furthermore, the results do not appear to be sensitive to the number of portfolios.

## 3.2 Portfolio Returns

Options are levered claims on the underlying stock. As a result of their embedded leverage, they tend to have loadings on systematic risk factors that are much larger than those of the underlying stock. It is very common for options to have market betas up to twenty times that of the underlying. This leverage effect can lead to very skewed returns on options. Highly volatile and skewed distributions are not well suited to estimating linear pricing kernels because a linear SDF is typically not able to capture such extremes. This fact makes linear factor models and the linear stochastic discount factor they imply a poor tool for analyzing raw option returns.

The embedded leverage of options further reduces the effectiveness of standard cross-sectional asset pricing techniques by rendering factor loadings less stable. In the Black-Scholes-Merton world, loadings of options on any risk factor are approximately equal to the loading of the underlying on the factor scaled by the leverage of the option. The leverage of each option is a function of instantaneous volatility of the underlying which presumably is correlated with volatility of the market. As such the correlation of an option with a risk factor changes with market volatility. This means that even if one forms portfolios of options, the portfolio loadings on risk factors will be sensitive to large changes in volatility. Cross-sectional regressions will thus be sensitive to the instability of portfolio factor loadings.

Forming portfolios of option returns helps to dampen the effect of outliers and thus reduces skewness and excess kurtosis. It also mitigates the problem of the sensitivity of factor loadings to changes in volatility by dampening the effect for those options whose factor loadings are the most sensitive to volatility. Leverage adjusting returns further reduces the effect of each problem. In a world where the Black-Scholes-Merton model holds perfectly, continuously adjusting each option according to its implied leverage will completely solve both problems. As long as the SDF projected onto the space of stock returns can be adequately estimated by a linear model, continuous leverage adjustment renders linear factor models capable of pricing options. Given the well-documented shortcomings of the Black-Scholes-Merton model and the fact that it is impossible to adjust leverage in continuous time, the best we can hope to do with this leverage adjustment is to approximately correct both problems.

The Black-Scholes-Merton implied leverage of an option is given by the elasticity of the option price with respect to the underlying stock's price,

$$\omega_{j,i,t}^{BSM} = \Delta_{j,i,t} \frac{S_{i,t}}{P_{j,i,t}},$$

where $\Delta_{j,i,t}$ is the time $t$ Black-Scholes-Merton option delta for option $j$ on stock $i$, $S_{i,t}$ is the price of the underlying stock and $P_{j,i,t}$ is the price of the option. Table 2 gives summary statistics for the Black-Scholes-Merton implied leverage of option contracts in the sample. In order to leverage-adjust the returns, I calculate the gross returns to investing $(\omega_{j,i,t}^{BSM})^{-1}$ dollars in option $j$ and $1 - (\omega_{j,i,t}^{BSM})^{-1}$ in the risk free rate. Since $\Delta$ is negative for puts and positive for calls, this amounts to a short position in puts and long position in calls. Leverage adjusted returns on the individual options are thus a linear combination of the returns on the risk free rate and returns calculated in Equations (1) and (2). Leverage adjustment is done at the beginning of the holding period, when the position is opened. Thus the leverage-adjusted returns are the returns to a portfolio composed of an option and the risk-free rate where the weight in the option is inversely related to its leverage. Unlike Constantinides, Jackwerth, and Savov (2013), I hold the portfolio fixed over time and do not re-adjust leverage as the option's leverage evolves over time. A trading strategy with daily adjustment would incur very high transaction costs since the costs of buying and selling options is generally much higher than the cost incurred when buying and selling more liquid securities. Therefore, in order to replicate a more feasible trading strategy, I create portfolios that do not change over the course of the holding period. The obvious trade off is that these portfolios will not be as free of excess kurtosis and skewness as they would be in the case of daily rebalancing.

The majority of papers in the empirical option pricing literature examine delta-hedged returns in order to study profitability of trading strategies where investors have taken a delta-hedged position in options. The risks whose prices are estimated using delta-hedged option returns like those in Duarte and Jones (2007) are risks *orthogonal* to the underlying asset. In this paper I examine the price of *total* volatility risk because this is the risk estimated from the cross-section of equity returns. It is also the risk whose premium is implicitly estimated by looking at the difference between risk-neutral and physical moments of the underlying asset as in Carr and Wu (2009) and Driessen, Maenhout, and Vilkov (2009). Since the purpose of this paper is to resolve the apparent discrepancy between prices of *total* volatility risk in options and equity, I do not delta hedge option returns.

Finally, to compute portfolio returns within each of the 36 portfolios, I weight the leverage-adjusted returns. In order to facilitate the comparison between the underlying stock returns and portfolios of option returns, I weight the options by the market capitalization of the underlying stock. This is standard practice in the equity pricing literature.

### 3.3 Summary Statistics

Table 3 gives summary statistics for the 36 value-weighted option portfolios. Panel A reports the annualized percentage mean returns of each of the 36 portfolios over the 200 months ranging from January 1997 through August 2013. The mean of the call portfolios is increasing in implied volatility risk premium while the mean of the put portfolios tends to decrease progressing from the lowest implied volatility premium, IVP1 to highest IVP6. Recall however that puts have a negative $\Delta$ and hence negative leverage, so the put portfolios are actually portfolios of short positions in the option. Therefore, long positions in the put portfolios earn increasing mean returns as a function of IVP. The dispersion in mean returns is much larger for the puts than calls but in all cases except ITM calls, the difference between mean returns in IVP1 and IVP6 is very large. As has been shown in the literature (see e.g. Coval and Shumway (2001)), selling puts is very lucrative because investors are willing to pay a premium to use puts as a hedge against large losses, so the large returns in the put portfolios is not surprising.

High levels of returns for puts and decreasing mean put returns as a function of moneyness are consistent with economic theory. The call portfolios however, exhibit increasing mean returns as a function of moneyness. As shown by Coval and Shumway (2001), if stock returns are positively correlated with aggregate wealth and investor utility is increasing and concave, then returns on European call options should be negatively sloped as a function of strike prices. While the options used in this paper are American, I have removed options that are likely to be exercised early so reasoning similar to that in Coval and Shumway (2001) should be applicable here. This is not the first paper to document this pattern in mean returns of equity call options; Ni (2008) documents this puzzle. She shows that considering only calls on stocks that do not pay dividends and hence should never be exercised early, this pattern still shows up in the data. Furthermore, the pattern is very robust to different measurements of returns and moneyness. The explanation proposed by Ni is that investors in OTM call options have preferences for idiosyncratic skewness for which they are willing to pay a premium in OTM calls.

Panel B reports annualized return volatility of each value-weighted option portfolio in percent. Volatility is monotonically decreasing in moneyness for the put portfolios. For the call portfolios the pattern is less clear. We also see that volatility is higher for the put portfolios than for the calls. Panels C and D report monthly measures of skewness and kurtosis for each portfolio. As can be seen in Figure 6, the put portfolios are negatively skewed while the calls are positively skewed. Furthermore, the magnitude of the skewness is highest in OTM options and tends to decrease monotonically in moneyness. Similarly, kurtosis is largest in OTM options and smallest in the ITM options, with a monotonic, decreasing pattern in moneyness. The purpose of forming leverage-adjusted portfolios of option returns is to reduce excess skewness and kurtosis, thus rendering portfolio returns nearly normally distributed. While the skewness measures are not equal to zero as one would ideally like to have, they are much smaller in magnitude than the skewness of raw option returns. For example, the absolute value of skewness for the empirical distribution of raw returns on all calls and puts used to form the portfolios are on average 4.769 and 6.263 respectively. Return kurtosis is reduced even more dramatically by forming the leverage-adjusted portfolios. The normal distribution has a kurtosis of 3. The kurtosis of the leverage-adjusted portfolios ranges from 3.763 to 14.615. The kurtosis of raw realized option returns of the options dwarfs that of the leverage-adjusted portfolios. This is most noticeable in the OTM options. The average kurtosis of the empirical distribution of raw returns on OTM calls is 44.297 while that of the OTM puts is 83.697. This means that forming portfolios of leverage adjusted returns reduces kurtosis by nearly 90% in OTM puts and 75% in OTM calls. That is, the shape of the tails of the empirical distribution of the OTM option portfolios is much closer to the that of a normal distribution than are the tails of the empirical distribution of raw returns on OTM options.

Panel E shows the CAPM betas for each portfolio. Recall that the put portfolios are actually short puts. This is why the betas reported for the puts are positive. Betas are monotonically increasing in moneyness for the calls and for the most part decreasing in moneyness for the puts. The betas on the calls are below one while the betas on the puts are mostly above one. Comparing these with the CAPM betas on the stock portfolios shown in Table 4 gives an indication of the leverage reduction achieved by leverage adjusting the returns in the option portfolios. It is quite common for options on individual stocks to have Black-Scholes-Merton implied leverages with magnitudes in excess of 20. If an option on a stock has an implied leverage of 20, then in the Black-Scholes-Merton world, for any risk factor, the beta of the option on that risk factor will be 20 times that of the underlying stock. In the case of the put portfolios, the CAPM betas are magnified by roughly 15% above those of the corresponding stock portfolios in Panel A of Table 4. In the case of calls, the betas are reduced by about 25% on average. In both cases this suggests a fairly low level of implied leverage in the options.

Panel F of Table 3 reports betas on systematic volatility in the two factor model

$$R_{i,t} = \beta_{M,i} MKT_t + \beta_{\Delta VIX,i} \Delta VIX_t + \epsilon_{i,t}, \tag{3}$$

where $MKT_t$ denotes time $t$ excess returns on the market and $\Delta VIX_t$ denotes first differences in the VIX index. The factors used to proxy for market returns and volatility innovations are formed as described in Section 2.4. The volatility betas of call portfolios are much smaller in magnitude than the volatility betas of the put portfolios. Half of the call portfolios betas are statistically significant at the 5% level. On the other hand, all of the volatility betas except that of the ITM IVP6 portfolio are highly significant. The average t-statistic of the put portfolios' volatility betas is $-4.33$, while that of the call portfolios is only 1.36. The fact that the puts appear to load so much more on the volatility factor suggests that if systematic volatility is indeed priced in equity options, the premium is more likely to be evident in the puts than the calls. Again, since the put portfolios are actually short puts, the loadings on volatility are negative. In both call and put portfolios, the *magnitude* of volatility betas decreases monotonically in moneyness.

Table 4 reports summary statistics for the stock portfolios. The portfolios are comprised of all CRSP stocks over the 200 months ranging from January 1997 through August 2013. The columns of each panel in the table represent sorts according to betas on market excess return over the previous month of daily data. Rows represent sorts according to loadings on volatility innovations. Panel A reports post formation value-weighted mean returns. For the most part, the post ranking mean returns are higher for the high market beta than the low market beta group. Mean returns to the portfolios are generally decreasing in loadings on the volatility factor as one would expect given that stocks with higher loadings on the VIX act as a hedge agains high volatility states and investors are thus willing to pay a premium for these stocks. The monotonicity in mean returns along the volatility loading dimension is not particularly strong. This is due to the fact that the formation period is only a month long.

Panel B reports annualized percent volatility. There is clear heteroskedasticity between the two market loading bins with the higher market-loading stocks having substantially higher volatility. Skewness is negative for all portfolios and tends to be larger in magnitude for the low market beta stocks than for the high beta stocks. The stock portfolios are less skewed than the option portfolios but the difference is not very dramatic. Similarly, the kurtosis of the stock portfolios is slightly smaller than the option portfolios except in the case of OTM puts where the kurtosis is most extreme. Figure 6 plots the histograms of realized returns for each of the six put/call and moneyness bins as well as the realized returns of all puts and all calls separately and all ATM options. Over each is the kernel density estimate of the empirical return distribution of the stock portfolios. One can see from the figure that skewness and variance of the option portfolios is not very different from that of the stocks except perhaps in the case of the OTM calls.

The post ranking CAPM betas of the stock portfolios are much larger for the stocks with large formation period betas suggesting that stocks' covariation with the market is fairly stable. On the other hand, Panel F shows that the post-ranking volatility betas do not exhibit a clear monotonic pattern. This indicates that at least with the one month formation window, stocks' loadings on innovations in the VIX are less stable.

# 4 Pricing Kernel Estimation

In this section I test a number of specifications of pricing kernels to assess the importance of volatility for the SDF projected onto the space of option returns. Throughout this section I use the Generalized Method of Moments of Hansen (1982) and Hansen and Singleton (1982) to perform the asset pricing tests. Since the tests combine various portfolios of options as well as stocks, using GMM circumvents any problems that may arise due to heteroskedasticity across asset classes or moneyness-put/call bins that are shown to exist in Tables 3 and 4. An additional advantage of the GMM methodology over regression-based cross-sectional tests like Fama and MacBeth (1973), is the fact that it avoids the error-in-variables problem associated with estimating risk factor loadings in time-series regressions which are subsequently used as independent variables in the cross-sectional regression. This errors in variables problem is particularly glaring in the case of option returns. If one uses individual options as test assets and computes returns to the value of the option at multiple times over the course of the option's lifetime, then any changes in leverage of the option will result in changes in factor loadings in time series regressions. Furthermore, the most liquid options are short dated, meaning that time-series regressions on option returns used in the first step of a procedure like Fama-MacBeth cannot be estimated with a very long time series.

The use of GMM coupled with the option portfolios described in Section 3 allows me to circumvent the errors in variables problem. GMM estimation does not require test asset returns to be iid conditional on risk factors. All we need is for our time series of portfolios to be stationary and ergodic.[9]

[9]In an unreported test, all but one of the 36 option portfolios described in Section 3 were able to reject non-stationarity at the 1% level using an the Augmented Dickey-Fuller test for non-stationarity. The one

## 4.1 GMM specification

In order to investigate the importance of market-wide stochastic volatility in the cross-section of option returns, I apply the GMM methodology of Hansen and Singleton (1982) to various specifications of a linear pricing kernel. The specifications include factors commonly used in the empirical asset pricing literature. In this sense, the models used in this paper are directly comparable to some of the most well known reduced form models used to study the cross-section of stock returns. I augment the models with the volatility factor in order to assess the importance of market-wide volatility in the SDF.

In addition to factors studied widely in the classical asset pricing literature, I include factors meant to capture market jump risk and market volatility jump, both of which are commonly included in theoretical option pricing models.[10] I include additional factors meant to capture extreme movements in the market that have been shown to perform well in pricing the cross-section of stock returns. All of these additional factors track extreme movements in the market and are meant to control for the fact that volatility can be difficult to distinguish from downturns in the market level or large changes in the market level.[11]

For each specification of the pricing kernel, I use the two step optimal GMM to estimate the prices of risk associated with each factor. The first stage estimation uses the identity weighting matrix. In the second stage estimation the weighting matrix is set equal to the inverse of the covariance matrix estimated from the first stage. I estimate the weighting matrix using the Newey and West (1987) spectral density estimator with 6 lags. As a robustness check I also run the same set of tests with a one-step GMM using the identity weighting matrix and also the one-step GMM using the weighting matrix of Hansen and Jagannathan (1997). In both cases the results are similar to those reported in this section. The volatility factor is significant at the 5% level in all specifications with both versions of the single-step GMM and the point estimates are very similar to those obtained with the

portfolio that was not able to reject at the 1% level did reject at the 10% level and the GMM estimation results of this section are not substantially changed by removing this single portfolio.

[10] See for example Pan (2002) and Eraker, Johannes, and Polson (2003).

[11] See Bates (2012) for a discussion of difficulties related to disentangling volatility from large changes in market level.

2-step GMM.

In each specification of the pricing kernel $M$, the first $N$ moment restrictions in the GMM test with $N$ test assets are given by

$$\mathbb{E}\left[M_t R_{j,t}\right] - 1 = 0, \tag{4}$$

for $j = 1, 2, ...., N$, where $R_{j,t}$ denotes the time $t$ gross return of portfolio $j$. The final moment condition which is implied by the risk-free rate is given by

$$\mathbb{E}\left[M_t\right] - \frac{1}{R^f} = 0, \tag{5}$$

where $R^f$ denotes the risk-free rate.

## 4.2 Linear Pricing Kernels

In this section I restrict our attention to linear pricing kernels of the form

$$M_t = a + b' f_t,$$

where $f$ is a vector of risk factors, $b$ is a fixed vector of prices of risk and $a$ is a constant.

Tables 6, 7, 8 and 9 report the results for five specifications of the linear pricing kernel. The first is the single factor model with only the volatility factor. The second and third models are respectively the standard CAPM and the CAPM augmented with volatility. Model four is the Fama-French/Carhart four factor model and the fifth model is the volatility-augmented version of model four. For each model I report point estimates of the coefficients with t-statistics in parentheses. The final two columns of each table report the J-statistic and associated p-value as well as the Hansen-Jagannathan distance which measures the distance between the implied stochastic discount factor and the set of feasible discount factors.

Table 6 reports results of the tests using all 36 option portfolios. The coefficient on the volatility factor is positive and very significant in each specification. A positive coefficient in the SDF implies that investors' marginal rates of substitution are increasing in volatility.

This means that investors are willing to pay a premium for assets that covary positively with innovations in volatility. In other words the price of volatility risk is negative. For both the CAPM and the four factor model, adding volatility substantially reduces the J-statistic and the Hansen-Jagannathan distance measure, indicating that the model fits the data much better with the volatility factor than without.

Data filters are implemented to remove illiquid options and I only consider options on S&P 500 constituents in order to avoid results driven by illiquid options. In order to further alleviate any concerns about illiquidity driving the results, I examine just the ATM option portfolios separately as these are the most liquid options according to trading volume. Table 7 reports the results which are quite similar to the tests with the full set of option portfolios. The volatility factor is always positive and significant and given the fact that we only have twelve test assets, the significance is very strong. In each specification, the model fit is substantially improved with the addition of the volatility factor.

Table 9 reports the pricing kernel estimates for the ATM options and the 12 stock portfolios combined. If volatility is a priced risk factor in the SDF, then the projection of the SDF onto the combined space of stocks and options should also have a positive, significant coefficient. This is confirmed in Table 9. It is worth noting that for the combined stock portfolios and ATM option portfolios, the reduction in J-statistics due to adding the volatility factor are very small. However, the Hansen-Jagannathan distance is substantially reduced. In the case of the SDF projected onto the space of stock returns only, Table 8 shows that the fit of the four-factor model improves with the addition of the volatility factor but the two-factor model actually fits worse with the addition of volatility. The volatility coefficient's point estimates for both the stock portfolios as well as the combined stock and ATM option portfolios are well below the point estimates for the full set of option portfolios.

The takeaway from Tables 6, 7, 8, 9 is a clearly priced systematic volatility risk factor in option returns. To assess the economic magnitude of the volatility premium one can easily use the coefficient in the SDF to calculate $\lambda^{VOL}$, the implied market price of the the volatility risk. $\lambda^{VOL}$ is equivalent to the prices of risk typically estimated in the second step of Fama-

MacBeth regressions. In the case of the full model (model 5), the market price of volatility, $\lambda^{VOL}$ is equal to -4.13% per month or -62.5% annualized. We can get a sense of how much of the difference in mean returns of the OTM puts and ITM puts is driven by volatility risk by comparing the average volatility betas for each group. For model 5, the average volatility betas for OTM puts and ITM puts are $-0.7042$ and $-0.3482$ respectively. Exposure to aggregate volatility therefore accounts for $(-0.70 - (-0.35)) \times -4.13\% = 1.47\%$ monthly or 19% annualized spread in returns between ITM and OTM puts. For the calls the average OTM beta is 0.238 and the average ITM beta is $-0.013$. Exposure to aggregate volatility therefore accounts for $(-0.013 - 0.238) \times -4.13\% = 1.37\%$ monthly or 17.7% annualized spread in returns in the calls. Thus the volatility premium is economically significant as well as statistically significant. It is also worth noting that the implied price of risk, $-4.13\%$ per month is 18% larger than the $-3.49\%$ price of risk estimated in Chang, Christoffersen, and Jacobs (2013) using stocks.

In an unreported robustness check, I run all of the tests with the same portfolio sorts but weight returns by option open interest rather than stock market capitalization. The results are similar. Volatility is always significant at the 5% level and the point estimates are similar to those reported in Tables 6, 7, 8, 9.

## 4.3 Exponentially Affine Pricing Kernels

In order to check that the linear form assigned to our pricing kernel is not responsible for the strong significance of the market-wide volatility factor, I test the same set of CAPM and Fama-French-Carhart factors augmented with volatility using an exponentially affine pricing kernel instead of a linear pricing kernel. Whereas standard asset pricing models assume a linearized SDF, the exponentially affine pricing kernel is closer to the kernel derived by hypothesizing a utility function for a representative investor and then solving for the marginal rate of substitution. For an investor with CRRA utility, the SDF can be expressed as

$$M_t = \beta \left( \frac{C_{t+1}}{C_t} \right)^{-\gamma},$$

where $C_t$ denotes time $t$ consumption, $\gamma$ denotes the coefficient of relative risk aversion and $\beta$ denotes the investors discount rate. By taking the exponential of the log of the pricing kernel this can be transformed to the exponentially affine form

$$M_{t+1} = e^{log\beta - \gamma log\frac{C_{t+1}}{C_t}}.$$

I use an exponentially affine pricing kernel which assumes a similar form,

$$M_{t+1} = e^{a+b'f_{t+1}}, \tag{6}$$

where $b$ is a deterministic vector of coefficients and $f$ is a vector of risk factors. The log-utility CAPM is a special case of the SDF in Equation (6) where $a = 0$, $b = -1$, $f = logR^W$ and $R^W$ is return on the wealth portfolio. The exponentially affine framework is better suited for analyzing skewed payoffs like options as it does not rely on linear approximations of the functional form of investors' marginal rates of substitution. Continuous time versions of exponentially affine pricing kernels are commonly used in structural option pricing models, where the factors are typically specific to the underlying asset as opposed to systematic factors.

Tables 10, 11, 12 and 13 report the results of GMM tests using the pricing kernel defined in Equation 6 with the same set of factors from Tables 6, 7, 8, 9.[12] The results again show that market-wide volatility is a significantly priced factor in the cross-section of option returns. The point estimates cannot be directly compared to those in the linear models. However, the volatility factor is estimated to be significantly positive. Table 10 reports the results from a one-step GMM estimation where the weighting matrix is set equal to the identity matrix. This greatly reduces the power of the test but is meant to allay any concerns about unstable inversion of the weighting matrix in nonlinear GMM estimation when the number of time series observations is not very large compared to the number of

[12] I also test the exponentially affine models with the non-orthogonalized volatility factor. I do this because the orthogonalization is linear with respect to market excess returns and I want to be sure that the linear nature of the orthogonalization is not responsible for the results in a nonlinear model. The results for the coefficient on the volatility factor were virtually unchanged.

cross-sectional observations.[13] Using all 36 option portfolios, the volatility factor is signifiant at the 5% level for the two models containing the market factor. In the single factor model the volatility factor is only significant at the 10% level. Given that the combination of a single step GMM and a non-linear model substantially reduces the power of the test, the fact that the volatility factor is still significant can be regarded as strong evidence in favor of the volatility factor.

Tables 11 and 13 give the results of the tests with only the ATM options and the combined portfolios of ATM options and the 12 stock portfolios. In all specifications the volatility factor is very significant. In the case of the test with only ATM options, including the volatility factor drastically reduces the J-statistic and the Hansen-Jagannathan distance, especially in the case of the 4-factor model. The results are not so strong when the ATM options are combined with the 12 stock portfolios in Table 13, however the volatility factor is still significant in all specifications, indicating that market-wide volatility plays an important role in the SDF projected onto the joint space of stock and option returns. This holds true despite the fact that for the stock portfolios alone, there is little evidence that the volatility factor is significant in Table 12. This is important for two reasons. First, it indicates that we have more power to estimate the role of market-wide volatility in the SDF when using options than using the same number of stocks portfolios. Comparing the 12 ATM option portfolios with 12 stock portfolios sorted in a way that has been the most successful thus far in the literature at showing a significant volatility factor, it is clear that the option portfolios are a more powerful set of test assets. Second, even if the SDF projected onto one space shows the volatility factor to be statistically insignificant, it is entirely possible that the factor is still significantly priced in the SDF. It may just be the case that the space of stock returns is orthogonal to the volatility factor in the SDF while the space of option returns is not orthogonal to the factor. If this is the case, we still expect to find that when estimated from returns on the joint space of stock and option returns, the factor should be significant as we find in Table 13.

[13] See Ferson and Foerster (1994) and Cochrane (2005) for discussions about GMM and small sample properties.

## 4.4 Pricing Kernels with Tail Risk

As first noted by Black (1976), volatility of the market is negatively correlated with the market's level. Table 5 shows that in the sample period 1997 through 2013, monthly innovations in the VIX and excess market returns are highly negatively correlated. This is the reason for using orthogonalized VIX innovations in the analysis throughout the paper. More recently Bates (2012) discusses the difficulty of separating changes in volatility from jumps. A number of papers have also shown that the risk neutral distribution of stock indices exhibit higher volatility, more negative skewness and have heavier tails than their corresponding physical distributions.[14] This indicates that option prices reflect premia for skewness and kurtosis as well as volatility. Furthermore, Bates (2000), Pan (2002) and Eraker, Johannes, and Polson (2003) have shown that jump risk tends to increase during times of higher market volatility. Taken together, all of these empirical regularities suggest that the risk premium attributed to market-wide volatility in our earlier tests may actually be due to fears of tail events. In this section I include additional factors in specifications of the SDF in order to control for the possibility of tail risk driving the significant volatility premium documented thus far. Tables 14, 15, 16 and 17 give results of linear models for the SDF with additional factors described in Section 2.4.

Tables 14 and 15 report results for test assets comprised of all 36 option portfolios and the ATM portfolios respectively. The clear result from these two tables is that volatility risk carries a significant, positive coefficient (and hence a negative price of risk) even when we control for tail risk. While some of the tail-risk factors appear to be significant in a number of the specifications, volatility is the only factor that is significant in all specifications in both tables. In Table 14, with all 36 option portfolios as the test assets, downside risk also appears significant and skewness is significant at the 10% level. However, in Table 15 where the test assets are the 12 ATM portfolios, neither is significant. This is likely to be at least partially attributable to the fact that we have a small number of test assets and thus less cross-sectional variation. However, volatility is clearly significant even with the small number

[14] See Jackwerth and Rubinstein (1996), Jackwerth (2000) and Bakshi, Kapadia, and Madan (2003).

of test assets and the additional controls for tail risk. It is also worth noting that the jump factor does not appear to be significantly priced even though jumps are often modeled in option returns. However, the jumps included in theoretical option pricing models are jumps in the underlying asset as opposed to market-wide jumps. Of course in the case of index options where the relation between jumps and option prices have been most studied (see for example Pan (2002) and Eraker, Johannes, and Polson (2003)), on cannot distinguish between market-wide risks and risks inherent only in the underlying asset.

Table 16 reports the results for the stock portfolios test assets. In this set of tests the volatility factor remains marginally significant at best. This could largely be due to the fact there is a small number of test assets. However, when compared to the 12 ATM option portfolios, it is clear that the volatility factor is much more prominent in the options than in the stock portfolios. In Table 17 where stocks and ATM options are the combined test assets, volatility is again very significant. Here skewness and downside risk are also significant.

The results of this section indicate that not only is market-wide volatility a significant risk factor in the cross-section of individual option returns, but it is distinct from market-wide tail risk. Taken together with tests in the previous sections this suggests that volatility is a very robustly priced risk factor in the cross-section.

# 5 Likelihood Ratio-Type Tests

In this section I test whether the prices of risk estimated using options differs from those estimated using the underlying stocks. The tests I use are special cases of those described in Andrews (1993). They are also known in the econometrics literature as likelihood ratio-type tests for GMM models. These tests combine stock and options data in restricted and unrestricted GMM tests and compare the resulting objective functions. In this way, the intuition behind the tests is similar to likelihood ratio tests. Of course the difference is that in this setting we have not specified a parametric likelihood function. Here, as in the previous section, I use GMM because I estimate models that simultaneously use stock portfolios and different option portfolios to estimate models. Tables 3 and 4 demonstrate the need for taking into account possible heteroskedasticity across assets.

Similar to likelihood ratio tests, the GMM likelihood ratio-type test compares the value of an objective function under the null hypothesis to its value under an alternative hypothesis. For the purpose of testing prices of risk in two markets, the comparison is made between models that fix the coefficients on risk factors to be the same in the option and equity pricing kernels and those that relax this assumption. I perform the tests by relaxing the assumption on the volatility factor and comparing the resulting unrestricted GMM objective function to the restricted objective function. Namely, the null hypothesis is

$$H_0 : b^S_{VOL} = b^O_{VOL}$$

where $b^S_{VOL}$ and $b^O_{VOL}$ are the prices of risk in the stock and option markets respectively.

For each proposed model, I estimate the restricted version by pooling stock portfolios together with option portfolios so that the test assets are a mix of the 12 stock portfolios and 12 ATM option portfolios. The results of estimating the restricted models are given in Table 9. For each model I test the restriction by relaxing $H_0$ and comparing the resulting fit to the corresponding model fit in the restricted model.

Since the tests compare GMM objective functions with and without a linear restriction, one needs to be sure that the difference in objective functions is not driven by the weighting matrix but is driven only by differences due to relaxing the restriction on a given factor. I therefore use the second stage weighting matrix from the restricted model estimation to estimate the unrestricted model in a single step GMM. This also ensures that the test statistic has a well defined asymptotic distribution. In particular, the test statistic has the asymptotic distribution given by

$$LR_{GMM} = T\left(m(\hat{\theta}_R)'W(\hat{\theta_R})m(\hat{\theta}_R) - m(\hat{\theta}_U)'W(\hat{\theta_R})m(\hat{\theta}_U)\right) \rightarrow \chi^2_1, \tag{7}$$

as $T \rightarrow \infty$, where $T$ denotes total number of observations, $\hat{\theta}_R$ and $\hat{\theta}_U$ denote estimated vectors of prices of risk under the restricted and unrestricted models respectively and $m(\hat{\theta}_R)$ and

$m(\hat{\theta}_U)$ denote empirical means of moment restrictions under the restricted and unrestricted models.

Table 18 gives the test statistics and corresponding p-values for each likelihood ratio-type test. Rows represent the models used for each test. Columns represent the variable whose price of risk is being tested. Of the three baseline models testing the volatility factor, one shows a significant difference between the restricted and unrestricted model at the 5% level and the remaining two give significant test statistics at the the 10% level. This suggests that the price of systematic volatility risk is not necessarily the same in the equity and option markets. Although these results suggest that there may be difference in the prices of volatility risk between the two markets, the difference is likely to fall within no-arbitrage bands as it is well known that no-arbitrage option price ranges can be fairly wide.[15]

The fact that there is a difference between prices of volatility risk in the stocks and put options is akin to there being a significant price of risk in delta-hedged returns of put options. Whereas delta hedged options look directly at the option with the risk due to the underlying subtracted off, the results here look at the difference in prices estimated from options and stocks separately. These are two similar ways of addressing the same question; Is there significantly priced volatility risk inherent in option contracts that is not due solely to the underlying stock? The fact that I find a positive difference between implied prices of risks suggests the answer is yes. It further provides evidence that options are not redundant securities.

# 6 Conclusion

Volatility is generally accepted as playing an important role in determining prices of options. The evidence of a volatility premium in the index options market is well documented. In addition, the growing literature on individual stock option returns is largely comprised of papers examining volatility characteristics and their relation to returns on options. The well documented differences in the volatility and variance risk premia between index options

[15]See Figlewski (1989) for example.

and individual stock options (see Driessen, Maenhout, and Vilkov (2009) and Bakshi and Kapadia (2003a)) suggests that the volatility risk premium inherent in index options may not necessarily translate to a similar premium existing in the cross-section of individual options. In fact, Duarte and Jones (2007) find that volatility risk is not significantly priced unconditionally in the cross-section of individual option returns and Di Pietro and Vainberg (2006) find volatility risk has the opposite sign in the cross-section of synthetic variance swaps as in the cross-section of stock returns.

Until now evidence had suggested that market-wide volatility may not be priced in individual stock options. I find that there is strong evidence of a significant market-wide volatility risk factor in the pricing kernel for options on individual stocks. This factor is economically and statistically very significant. My results lend support to recent papers like Dittmar and Lundblad (2014), Boguth and Kuehn (2013), Campbell, Giglio, Polk, and Turley (2012) and Bansal, Kiku, Shaliastovich, and Yaron (2013) all of which suggest volatility is a priced state variable in the ICAPM sense. If volatility is a state variable in the ICAPM sense, it should be priced in the cross-section of individual options as well. The results of this paper thus make the make plausible the argument for volatility as a state factor.

I further find evidence that the price of market-wide volatility risk is greater in the the options than in the underlying stocks. This suggests that options are not redundant securities. Furthermore, it suggests that a potential reason for the existence of the option market may be as a market for hedging market-wide volatility risk.

# References


Adrian, Tobias, and Joshua Rosenberg, 2008, Stock returns and volatility: Pricing the short-run and long-run components of market risk, *The Journal of Finance* 63, 2997–3030.

Andrews, Donald WK, 1993, Tests for parameter instability and structural change with unknown change point, *Econometrica: Journal of the Econometric Society* pp. 821–856.

Ang, Andrew, Joseph Chen, and Yuhang Xing, 2006, Downside risk, *Review of Financial Studies* 19, 1191–1239.

Ang, Andrew, Robert J Hodrick, Yuhang Xing, and Xiaoyan Zhang, 2006, The cross-section of volatility and expected returns, *The Journal of Finance* 61, 259–299.

Bakshi, Gurdip, Charles Cao, and Zhiwu Chen, 2000, Do call prices and the underlying stock always move in the same direction?, *Review of Financial Studies* 13, 549–584.

Bakshi, Gurdip, and Nikunj Kapadia, 2003a, Delta-hedged gains and the negative market volatility risk premium, *Review of Financial Studies* 16, 527–566.

——— , 2003b, Volatility risk premiums embedded in individual equity options: Some new insights, *The Journal of Derivatives* 11, 45–54.

——— , and Dilip Madan, 2003, Stock return characteristics, skew laws, and the differential pricing of individual equity options, *Review of Financial Studies* 16, 101–143.

Bansal, Ravi, Dana Kiku, Ivan Shaliastovich, and Amir Yaron, 2013, Volatility, the macroeconomy, and asset prices, *The Journal of Finance*.

Bates, David S, 2000, Post-'87 crash fears in the s&p 500 futures option market, *Journal of Econometrics* 94, 181–238.

——— , 2012, Us stock market crash risk, 1926–2010, *Journal of Financial Economics* 105, 229–259.

Black, Fischer, 1976, {Studies of stock price volatility changes}, *Proceedings of the American Statistical Association, Business and Economic Statistics Section* pp. 177–181.

Boguth, Oliver, and Lars-Alexander Kuehn, 2013, Consumption volatility risk, *The Journal of Finance* 68, 2589–2615.

Bollen, Nicolas PB, and Robert E Whaley, 2004, Does net buying pressure affect the shape of implied volatility functions?, *The Journal of Finance* 59, 711–753.

Boyer, Brian H, and Keith Vorkink, 2014, Stock options as lotteries, *The Journal of Finance*.

Buraschi, Andrea, and Jens Jackwerth, 2001, The price of a smile: Hedging and spanning in option markets, *Review of Financial Studies* 14, 495–527.

Campbell, John Y, Stefano Giglio, Christopher Polk, and Robert Turley, 2012, An intertemporal capm with stochastic volatility, Discussion paper, National Bureau of Economic Research.

Cao, Jie, and Bing Han, 2013, Cross section of option returns and idiosyncratic stock volatility, *Journal of Financial Economics* 108, 231–249.

Carr, Peter, and Liuren Wu, 2009, Variance risk premiums, *Review of Financial Studies* 22, 1311–1341.

Chang, Bo Young, Peter Christoffersen, and Kris Jacobs, 2013, Market skewness risk and the cross section of stock returns, *Journal of Financial Economics* 107, 46–68.

Christoffersen, Peter, Ruslan Goyenko, Kris Jacobs, and Mehdi Karoui, 2011, Illiquidity premia in the equity options market, *Available at SSRN 1784868.*

Cochrane, John Howland, 2005, *Asset pricing* . vol. 41 (Princeton university press Princeton).

Constantinides, George M, Jens Carsten Jackwerth, and Alexi Savov, 2013, The puzzle of index option returns, *Review of Asset Pricing Studies* p. rat004.

Coval, Joshua D, and Tyler Shumway, 2001, Expected option returns, *The journal of Finance* 56, 983–1009.

Cox, John C, Stephen A Ross, and Mark Rubinstein, 1979, Option pricing: A simplified approach, *Journal of financial Economics* 7, 229–263.

Da, Zhi, and Ernst Schaumburg, 2009, The pricing of volatility risk across asset classes and the fama-french factors, Discussion paper, Working paper, Northwestern University.

Di Pietro, Vadim, and Gregory Vainberg, 2006, Systematic variance risk and firm characteristics in the equity options market, *Available at SSRN 858304*.

Dittmar, Robert, and Christian Lundblad, 2014, A simple consumption-based asset pricing model and the cross-section of equity returns, *Manuscript*.

Drechsler, Itamar, and Amir Yaron, 2011, What's vol got to do with it, *Review of Financial Studies* 24, 1–45.

Driessen, Joost, Pascal J Maenhout, and Grigory Vilkov, 2009, The price of correlation risk: Evidence from equity options, *The Journal of Finance* 64, 1377–1406.

Duan, Jin-Chuan, and Jason Wei, 2009, Systematic risk and the price structure of individual equity options, *Review of Financial Studies* 22, 1981–2006.

Duarte, Jefferson, and Christopher S Jones, 2007, The price of market volatility risk, *Unpublished working paper, University of Southern California*.

Eraker, Bjørn, Michael Johannes, and Nicholas Polson, 2003, The impact of jumps in volatility and returns, *The Journal of Finance* 58, 1269–1300.

Fama, Eugene F, and James D MacBeth, 1973, Risk, return, and equilibrium: Empirical tests, *The Journal of Political Economy* pp. 607–636.

Ferson, Wayne E, and Stephen R Foerster, 1994, Finite sample properties of the generalized method of moments in tests of conditional asset pricing models, *Journal of Financial Economics* 36, 29–55.

Figlewski, Stephen, 1989, Options arbitrage in imperfect markets, *The Journal of Finance* 44, 1289–1311.

Frazzini, Andrea, and Lasse H Pedersen, 2012, Embedded leverage, Discussion paper, National Bureau of Economic Research.

Garleanu, Nicolae, Lasse Heje Pedersen, and Allen M Poteshman, 2009, Demand-based option pricing, *Review of Financial Studies* 22, 4259–4299.

Goyal, Amit, and Alessio Saretto, 2009, Cross-section of option returns and volatility, *Journal of Financial Economics* 94, 310–326.

Hansen, Lars Peter, 1982, Large sample properties of generalized method of moments estimators, *Econometrica: Journal of the Econometric Society* pp. 1029–1054.

———, and Ravi Jagannathan, 1997, Assessing specification errors in stochastic discount factor models, *The Journal of Finance* 52, 557–590.

Hansen, Lars Peter, and Kenneth J Singleton, 1982, Generalized instrumental variables estimation of nonlinear rational expectations models, *Econometrica: Journal of the Econometric Society* pp. 1269–1286.

Jackwerth, Jens Carsten, 2000, Recovering risk aversion from option prices and realized returns, *Review of Financial Studies* 13, 433–451.

———, and Mark Rubinstein, 1996, Recovering probability distributions from option prices, *The Journal of Finance* 51, 1611–1631.

Lettau, Martin, Matteo Maggiori, and Michael Weber, 2013, Conditional risk premia in currency markets and other asset classes, Discussion paper, National Bureau of Economic Research.

Newey, Whitney K, and Kenneth D West, 1987, Hypothesis testing with efficient method of moments estimation, *International Economic Review* pp. 777–787.

Ni, Sophie X, 2008, Stock option returns: a puzzle, *Available at SSRN 1259703*.

Pan, Jun, 2002, The jump-risk premia implicit in options: Evidence from an integrated time-series study, *Journal of financial economics* 63, 3–50.

Vanden, Joel M, 2004, Options trading and the capm, *Review of Financial Studies* 17, 207–238.

Table 1: Options Sample

This table gives the number of option contracts considered in our sample for each of the six call/put and moneyness bins over the 200 month sample from January 1997 through August 2013. There are a total of 599,803 options in the filtered data.

**Number of Options**

| | OTM | ATM | ITM |
|---|---|---|---|
| Calls | 93,658 | 127,423 | 77,348 |
| Puts | 101,925 | 107,419 | 92,030 |

Table 2: Option Leverage Estimates

This table gives summary statistics for the Black-Scholes-Merton estimates of leverage in individual stock options in the sample.

**Option Leverage**

| | OTM | | ATM | | ITM | |
|---|---|---|---|---|---|---|
| | **mean** | **std dev** | **mean** | **std dev** | **mean** | **std dev** |
| Calls | 18.31 | 7.33 | 14.79 | 6.43 | 8.16 | 3.48 |
| Puts | -15.37 | 6.65 | -13.03 | 6.20 | -7.36 | 3.83 |

Table 3: Summary statistics for 36 value-weighted option portfolios

This table reports summary statistics for each of the 36 value-weighted option portfolios. Columns represent OTM, ATM or ITM calls and puts. Rows represent portfolios sorted by implied volatility premium (IVP) within each moneyness, option-type portfolio; IVP1 denotes portfolio with the smallest implied volatility premium while IVP6 represents the portfolio with largest implied volatility premium. Mean and volatility are reported in terms of annualized returns in percent. Skewness and kurtosis are measures of monthly holding period returns. The sample covers 200 months, from January 1997 through August 2013.

| | Calls | | | Puts | | | Calls | | | Puts | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | OTM | ATM | ITM | OTM | ATM | ITM | OTM | ATM | ITM | OTM | ATM | ITM |
| IVP | **A. Mean (%)** | | | | | | **B. Volatility (%)** | | | | | |
| IVP1 | 3.963 | 5.818 | 4.799 | 4.525 | 5.532 | 8.654 | 33.245 | 25.593 | 24.323 | 42.037 | 34.630 | 29.701 |
| IVP2 | 3.190 | 5.148 | 6.082 | 13.427 | 8.392 | 8.466 | 21.402 | 19.201 | 17.955 | 29.903 | 26.582 | 22.706 |
| IVP3 | 4.704 | 5.437 | 4.905 | 16.617 | 11.619 | 13.673 | 19.859 | 16.802 | 17.207 | 29.806 | 23.699 | 21.198 |
| IVP4 | 3.224 | 6.281 | 8.046 | 17.635 | 14.150 | 13.560 | 18.007 | 17.121 | 17.703 | 29.935 | 25.196 | 21.541 |
| IVP5 | -5.258 | 4.395 | 7.737 | 25.768 | 22.448 | 13.171 | 18.728 | 20.395 | 20.362 | 31.417 | 26.620 | 26.335 |
| IVP6 | -14.171 | -2.688 | 1.223 | 33.907 | 24.271 | 25.377 | 22.941 | 24.516 | 26.332 | 35.676 | 34.025 | 28.949 |
| | **C. Skewness** | | | | | | **D. Kurtosis** | | | | | |
| IVP1 | 5.777 | 2.611 | 1.759 | -3.343 | -2.330 | -1.544 | 9.153 | 6.475 | 4.980 | 11.108 | 9.838 | 6.732 |
| IVP2 | 2.073 | 1.291 | 0.315 | -3.144 | -2.227 | -1.306 | 9.630 | 6.677 | 3.935 | 10.415 | 10.161 | 6.168 |
| IVP3 | 1.993 | 0.760 | 0.163 | -3.520 | -1.902 | -1.374 | 8.812 | 4.162 | 3.763 | 12.348 | 8.491 | 6.034 |
| IVP4 | 1.083 | 0.481 | 0.015 | -3.706 | -2.085 | -1.496 | 4.008 | 3.572 | 4.009 | 14.615 | 9.633 | 7.398 |
| IVP5 | 1.669 | 0.925 | -0.152 | -3.057 | -2.210 | -1.334 | 7.677 | 5.343 | 4.423 | 11.579 | 11.809 | 7.446 |
| IVP6 | 1.708 | 0.775 | 0.030 | -2.248 | -1.478 | -0.929 | 7.936 | 4.741 | 4.073 | 7.201 | 8.092 | 6.448 |
| | **E. CAPM beta** | | | | | | **F. Volatility beta (2 factor model)** | | | | | |
| IVP1 | 0.874 | 0.886 | 0.943 | 1.589 | 1.456 | 1.292 | 0.545 | 0.445 | 0.238 | -0.610 | -0.394 | -0.191 |
| IVP2 | 0.612 | 0.726 | 0.779 | 1.228 | 1.179 | 1.063 | 0.306 | 0.180 | 0.012 | -0.616 | -0.421 | -0.240 |
| IVP3 | 0.622 | 0.650 | 0.766 | 1.174 | 1.089 | 0.988 | 0.197 | 0.145 | -0.053 | -0.663 | -0.419 | -0.321 |
| IVP4 | 0.571 | 0.720 | 0.816 | 1.231 | 1.123 | 1.009 | 0.232 | 0.109 | -0.050 | -0.697 | -0.521 | -0.374 |
| IVP5 | 0.571 | 0.796 | 0.913 | 1.247 | 1.150 | 1.130 | 0.072 | 0.123 | -0.072 | -0.851 | -0.635 | -0.441 |
| IVP6 | 0.617 | 0.925 | 1.073 | 1.194 | 1.338 | 1.180 | 0.225 | 0.092 | -0.038 | -0.720 | -0.657 | -0.451 |

Table 4: Summary statistics for stock portfolios

This table reports summary statistics for the stock portfolios formed according to the double sorting procedure. Where the first sort is by $\beta_M$, each stock's market beta. The second sort is by $\beta_{\Delta VIX}$, stock loading on changes in the VIX. Mean and volatility are reported in terms of annualized returns in percent. Skewness and kurtosis are measures of monthly holding period returns. The sample includes all CRSP stocks and covers 200 months, from January 1997 through August 2013.

**Stock Portfolios**

| | $\beta_M$ | | $\beta_M$ | |
|---|---|---|---|---|
| | (1) | (2) | (1) | (2) |
| $\beta_{\Delta VIX}$ | **A. Mean (%)** | | **B. Volatility (%)** | |
| (1) | 11.023 | 12.499 | 18.526 | 30.168 |
| (2) | 7.862 | 7.785 | 14.912 | 24.610 |
| (3) | 7.987 | 8.718 | 14.181 | 22.403 |
| (4) | 6.208 | 10.955 | 14.604 | 24.414 |
| (5) | 8.524 | 11.884 | 15.761 | 26.514 |
| (6) | 7.494 | 4.996 | 22.102 | 33.240 |
| | **C. Skewness** | | **D. Kurtosis** | |
| (1) | -0.660 | -0.908 | 4.918 | 6.458 |
| (2) | -1.078 | -1.208 | 4.936 | 7.320 |
| (3) | -1.429 | -0.861 | 7.186 | 7.094 |
| (4) | -1.354 | -0.232 | 7.263 | 5.967 |
| (5) | -1.126 | -0.431 | 5.801 | 5.600 |
| (6) | -1.307 | -0.416 | 6.504 | 5.115 |
| | **E. CAPM beta** | | **F. Volatility beta** | |
| (1) | 0.810 | 1.417 | -0.094 | -0.074 |
| (2) | 0.679 | 1.210 | -0.082 | 0.055 |
| (3) | 0.654 | 1.115 | -0.097 | 0.013 |
| (4) | 0.661 | 1.209 | -0.158 | 0.008 |
| (5) | 0.716 | 1.289 | -0.076 | 0.040 |
| (6) | 0.945 | 1.535 | -0.101 | 0.149 |

Table 5: Risk factor correlations

This table presents correlations between the risk factors examined in the paper. Construction of the factors is described in Section 2.4. The sample covers 200 months, from January 1997 through August 2013.

**Factor Correlations**

| | MKT | SMB | HML | Mom | VOL | $VOL^{\perp}$ | DS | Skew | Jump | VJ |
|---|---|---|---|---|---|---|---|---|---|---|
| MKT | 1.000 | | | | | | | | | |
| SMB | 0.304 | 1.000 | | | | | | | | |
| HML | -0.092 | -0.148 | 1.000 | | | | | | | |
| Mom | -0.348 | -0.048 | -0.305 | 1.000 | | | | | | |
| VOL | -0.777 | -0.218 | -0.002 | 0.284 | 1.000 | | | | | |
| $VOL^{\perp}$ | 0.000 | 0.028 | -0.117 | 0.022 | 0.630 | 1.000 | | | | |
| DS | 0.755 | 0.260 | 0.046 | -0.215 | -0.693 | -0.168 | 1.000 | | | |
| Skew | -0.213 | -0.079 | -0.114 | -0.040 | 0.339 | 0.275 | -0.673 | 1.000 | | |
| Jump | 0.480 | 0.224 | 0.141 | -0.119 | -0.456 | -0.131 | 0.674 | -0.602 | 1.000 | |
| VJ | -0.501 | -0.281 | -0.103 | 0.120 | 0.502 | 0.178 | -0.621 | 0.368 | -0.700 | 1.000 |

Table 6: Linear GMM Tests with 36 Option Portfolios

This table reports results of GMM tests of linear pricing kernels using all 36 options portfolios as test assets. Each row represents a model and columns represent factors included in the model. The point estimates are reported along with t-statistics in parentheses that are computed using Newey-West adjusted standard errors with 6-month lags. The final two columns give the J-statistic with corresponding asymptotic p-value in [brackets] and the Hansen-Jagannathan measure of distance from the space of valid stochastic discount factors.

**All 36 Options Portfolios**

| | intercept | MKT | SMB | HML | MOM | VOL | Jstat | HJ dist |
|---|---|---|---|---|---|---|---|---|
| (1) | 0.960 | | | | | 13.915 | 141.749 | 0.671 |
| | (40.172) | | | | | (5.399) | [0.000] | |
| (2) | 1.014 | -0.026 | | | | | 167.332 | 0.708 |
| | (92.067) | (-2.802) | | | | | [0.000] | |
| (3) | 0.962 | 0.015 | | | | 14.084 | 139.063 | 0.684 |
| | (38.065) | (1.303) | | | | (5.340) | [0.000] | |
| (4) | 1.081 | -0.019 | 0.941 | -250.931 | 9.075 | | 128.420 | 0.813 |
| | (13.471) | (-0.776) | (0.132) | (-3.244) | (2.666) | | [0.000] | |
| (5) | 0.991 | 0.006 | -0.153 | -55.246 | 3.260 | 17.536 | 73.384 | 0.668 |
| | (9.249) | (0.221) | (-0.024) | (-1.068) | (1.577) | (3.660) | [0.000] | |

Table 7: Linear GMM Tests ATM Portfolios

This table reports results of GMM tests of pricing kernels using the combination of 6 ATM put portfolios and 6 ATM call portfolios as test assets. Each row represents a model and columns represent factors included in the model. The point estimates are reported along with t-statistics in parentheses that are computed using Newey-West adjusted standard errors with 6-month lags. The final two columns give the J-statistic with corresponding asymptotic p-value in [brackets] and the Hansen-Jagannathan measure of distance from the space of valid stochastic discount factors.

**ATM calls and puts**

| | intercept | MKT | SMB | HML | MOM | VOL | Jstat | HJ dist |
|---|---|---|---|---|---|---|---|---|
| (1) | 1.021 | | | | | 15.982 | 16.714 | 0.324 |
| | (23.410) | | | | | (3.498) | [0.117] | |
| (2) | 1.010 | -0.019 | | | | | 26.991 | 0.388 |
| | (88.323) | (-1.528) | | | | | [0.005] | |
| (3) | 1.020 | -0.004 | | | | 15.718 | 17.123 | 0.322 |
| | (23.772) | (-0.212) | | | | (3.364) | [0.072] | |
| (4) | 1.182 | 0.003 | -14.857 | -179.253 | -0.950 | | 29.922 | 0.385 |
| | (14.540) | (0.098) | (-1.549) | (-2.348) | (-0.167) | | [0.000] | |
| (5) | 0.981 | -0.037 | 2.896 | 39.551 | -6.911 | 15.278 | 12.277 | 0.325 |
| | (15.764) | (-1.452) | (0.374) | (0.456) | (-1.674) | (2.288) | [0.092] | |

Table 8: Linear GMM Tests for Stocks

This table reports results of GMM tests of pricing kernels using the 12 stock portfolios as test assets. Each row represents a model and columns represent factors included in the model. The point estimates are reported along with t-statistics in parentheses that are computed using Newey-West adjusted standard errors with 6-month lags. The final two columns give the J-statistic with corresponding asymptotic p-value in [brackets] and the Hansen-Jagannathan measure of distance from the space of valid stochastic discount factors.

**Stock Portfolios**

| | intercept | MKT | SMB | HML | MOM | VOL | Jstat | HJ dist |
|---|---|---|---|---|---|---|---|---|
| (1) | 0.992 | | | | | 12.005 | 28.838 | 0.297 |
| | (20.707) | | | | | (2.011) | [0.002] | |
| (2) | 1.011 | -0.025 | | | | | 25.149 | 0.306 |
| | (74.416) | (-1.974) | | | | | [0.009] | |
| (3) | 1.008 | -0.047 | | | | 11.842 | 25.139 | 0.335 |
| | (25.507) | (-2.298) | | | | (1.768) | [0.005] | |
| (4) | 1.061 | -0.062 | 2.107 | -35.335 | -4.288 | | 17.426 | 0.363 |
| | (20.830) | (-2.045) | (0.449) | (-0.516) | (-1.284) | | [0.026] | |
| (5) | 1.006 | -0.024 | 9.532 | 139.756 | 5.472 | 11.644 | 9.287 | 0.312 |
| | (9.490) | (-0.534) | (1.127) | (1.017) | (0.939) | (1.513) | [0.233] | |

Table 9: Linear GMM Tests for Combined Stock Portfolios and ATM Options

This table reports results of GMM tests of pricing kernels using the 12 ATM option portfolios combined with the 12 stock portfolios as test assets. Each row represents a model and columns represent factors included in the model. The point estimates are reported along with t-statistics in parentheses that are computed using Newey-West adjusted standard errors with 6-month lags. The final two columns give the J-statistic with corresponding asymptotic p-value in [brackets] and the Hansen-Jagannathan measure of distance from the space of valid stochastic discount factors.

**12 ATM Options Portfolios and 12 Stock Portfolios**

| | intercept | MKT | SMB | HML | MOM | VOL | Jstat | HJ dist |
|---|---|---|---|---|---|---|---|---|
| (1) | 0.981 | | | | | 8.262 | 88.983 | 0.505 |
| | (57.297) | | | | | (3.165) | [0.000] | |
| (2) | 1.023 | -0.038 | | | | | 84.117 | 0.560 |
| | (63.870) | (-3.416) | | | | | [0.000] | |
| (3) | 0.994 | -0.030 | | | | 9.052 | 83.737 | 0.504 |
| | (44.188) | (-2.204) | | | | (3.000) | [0.000] | |
| (4) | 1.121 | -0.068 | 2.004 | -182.055 | -4.373 | | 74.183 | 0.607 |
| | (21.060) | (-3.891) | (0.517) | (-3.611) | (-1.868) | | [0.000] | |
| (5) | 1.031 | -0.046 | 0.801 | -95.068 | -1.797 | 9.460 | 73.399 | 0.523 |
| | (22.574) | (-2.588) | (0.234) | (-2.182) | (-0.945) | (2.828) | [0.000] | |

Table 10: GMM tests 36 option portfolios and Exponentially Affine SDF

This table reports results of GMM tests of exponentially affine pricing kernels using all 36 option portfolios. This is the only table in the paper that reports results for the 1-step GMM with an identity weighting matrix. I use this test instead of the 2-step GMM for this particular test in order to avoid problems associated with multiple-step GMM estimation of non-linear models when the time series of observations is not long compared to the number of test assets. Each row represents a model and columns represent factors included in the model. The point estimates are reported along with t-statistics in parentheses that are computed using Newey-West adjusted standard errors with 6-month lags. The final two columns give the J-statistic with corresponding asymptotic p-value in [brackets] and the Hansen-Jagannathan measure of distance from the space of valid stochastic discount factors.

**36 Options Exponentially Affine SDF**

| | intercept | MKT | SMB | HML | MOM | VOL | Jstat | HJ dist |
|---|---|---|---|---|---|---|---|---|
| (1) | -0.068 | | | | | 10.578 | 164.526 | 0.669 |
| | -(1.026) | | | | | (1.878) | [0.000] | |
| (2) | -0.000 | -0.035 | | | | | 167.256 | 0.680 |
| | (-0.045) | (-1.601) | | | | | [0.000] | |
| (3) | -0.063 | -0.002 | | | | 10.230 | 163.140 | 0.678 |
| | (-1.010) | (-0.072) | | | | (1.966) | [0.000] | |
| (4) | -0.075 | -0.012 | 1.034 | -57.061 | 4.521 | | 187.225 | 0.935 |
| | (-1.114) | (-0.516) | (0.119) | (-0.560) | (1.110) | | [0.000] | |
| (5) | -0.259 | -0.001 | 11.357 | 68.002 | 2.679 | 15.457 | 127.853 | 0.694 |
| | (-1.573) | (-0.036) | (1.196) | (0.962) | (0.774) | (2.748) | [0.000] | |

Table 11: GMM tests ATM option portfolios and Exponentially Affine SDF

This table reports results of GMM tests of exponentially affine pricing kernels using portfolios the 12 ATM option portfolios. Each row represents a model and columns represent factors included in the model. The point estimates are reported along with t-statistics in parentheses that are computed using Newey-West adjusted standard errors with 6-month lags. The final two columns give the J-statistic with corresponding asymptotic p-value in [brackets] and the Hansen-Jagannathan measure of distance from the space of valid stochastic discount factors.

**ATM Options Exponentially Affine SDF**

| | intercept | MKT | SMB | HML | MOM | VOL | Jstat | HJ dist |
|---|---|---|---|---|---|---|---|---|
| (1) | -0.089 | | | | | 12.518 | 19.560 | 0.330 |
| | -(1.767) | | | | | (2.999) | [0.052] | |
| (2) | 0.004 | -0.017 | | | | | 26.480 | 0.386 |
| | (0.677) | (-1.492) | | | | | [0.006] | |
| (3) | -0.085 | -0.002 | | | | 12.219 | 19.700 | 0.329 |
| | (-1.705) | (-0.117) | | | | (2.918) | [0.032] | |
| (4) | -0.341 | -0.036 | -22.754 | -303.736 | -12.031 | | 23.305 | 0.458 |
| | (-1.165) | (-1.767) | (-2.478) | (-4.172) | (-2.989) | | [0.003] | |
| (5) | -0.505 | 0.002 | -22.518 | -160.048 | -11.553 | 16.424 | 8.434 | 0.378 |
| | (-1.655) | (0.071) | (-2.272) | (-1.304) | (-2.069) | (2.379) | [0.296] | |

Table 12: GMM tests 12 Stock Portfolios and Exponentially Affine SDF

This table reports results of GMM tests of exponentially affine pricing kernels using the 12 stock portfolios. Each row represents a model and columns represent factors included in the model. The point estimates are reported along with t-statistics in parentheses that are computed using Newey-West adjusted standard errors with 6-month lags. The final two columns give the J-statistic with corresponding asymptotic p-value in [brackets] and the Hansen-Jagannathan measure of distance from the space of valid stochastic discount factors.

**12 Stock Portfolios**

| | intercept | MKT | SMB | HML | MOM | VOL | Jstat | HJ dist |
|---|---|---|---|---|---|---|---|---|
| (1) | -0.013 | | | | | -3.241 | 35.386 | 0.297 |
| | (-0.246) | | | | | (-0.232) | [0.006] | |
| (2) | -0.001 | -0.010 | | | | | 26.088 | 0.310 |
| | (-0.151) | (-0.632) | | | | | [0.073] | |
| (3) | -0.033 | -0.010 | | | | 7.535 | 25.454 | 0.347 |
| | (-0.420) | (-0.585) | | | | (0.814) | [0.062] | |
| (4) | -0.001 | -0.019 | 2.300 | -8.952 | -0.538 | | 25.046 | 0.367 |
| | (-0.028) | (-0.896) | (0.448) | (-0.123) | (-0.185) | | [0.034] | |
| (5) | -0.136 | -0.001 | 2.086 | 59.796 | 2.487 | 12.436 | 22.345 | 0.452 |
| | (-0.871) | (-0.048) | (0.361) | (0.859) | (0.650) | (1.445) | [0.050] | |

Table 13: GMM Tests with Exponentially Affine SDF
This table reports results of GMM tests of exponentially affine pricing kernels using portfolios the 12 ATM option portfolios combined with the 12 stock portfolios. Each row represents a model and columns represent factors included in the model. The point estimates are reported along with t-statistics in parentheses that are computed using Newey-West adjusted standard errors with 6-month lags. The final two columns give the J-statistic with corresponding asymptotic p-value in [brackets] and the Hansen-Jagannathan measure of distance from the space of valid stochastic discount factors.

**12 ATM Option Portfolios and 12 Stock Portfolios**

| | intercept | MKT | SMB | HML | MOM | VOL | Jstat | HJ dist |
|---|---|---|---|---|---|---|---|---|
| (1) | -0.053 | | | | | 6.301 | 85.832 | 0.516 |
| | (-2.947) | | | | | (2.184) | [0.000] | |
| (2) | 0.004 | -0.031 | | | | | 86.080 | 0.554 |
| | (0.643) | (-2.984) | | | | | [0.000] | |
| (3) | -0.046 | -0.019 | | | | 6.150 | 83.037 | 0.512 |
| | (-2.757) | (-1.733) | | | | (2.092) | [0.000] | |
| (4) | -0.036 | -0.043 | 4.907 | -120.448 | -2.315 | | 81.892 | 0.636 |
| | (-0.852) | (-3.061) | (0.958) | (-2.619) | (-1.116) | | [0.000] | |
| (5) | -0.105 | -0.031 | 4.972 | -97.279 | -1.535 | 8.349 | 67.327 | 0.556 |
| | (-2.146) | (-2.053) | (0.875) | (-1.794) | (-0.659) | (2.665) | [0.000] | |

Table 14: Linear GMM tests with Tail Risk

This table reports results of GMM tests of pricing kernels using all 36 options portfolios as test assets. Each row represents a model and columns represent factors included in the model. The point estimates are reported along with t-statistics in parentheses that are computed using Newey-West adjusted standard errors with 6-month lags. The final two columns give the J-statistic with corresponding asymptotic p-value in [brackets] and the Hansen-Jagannathan measure of distance from the space of valid stochastic discount factors.

**All 36 Options Portfolios**

| | intercept | MKT | SMB | HML | MOM | VOL | DS | SKEW | JUMP | VOL JUMP | Jstat | HJ dist |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| (1) | 1.069 | -0.052 | | | | 19.435 | 13.801 | | | | 63.047 | 0.670 |
| | (17.045) | (-1.965) | | | | (5.520) | (3.454) | | | | [0.001] | |
| (2) | 0.985 | -0.021 | | | | 13.326 | | -6.255 | | | 59.437 | 0.685 |
| | (11.144) | (-0.866) | | | | (3.033) | | (-1.709) | | | [0.003] | |
| (3) | 1.013 | -0.008 | | | | 17.739 | | | 0.645 | | 91.861 | 0.686 |
| | (19.489) | (-0.491) | | | | (5.570) | | | (0.606) | | [0.000] | |
| (4) | 1.115 | -0.020 | | | | 17.256 | | | | -6.920 | 95.473 | 0.686 |
| | (18.961) | (-1.076) | | | | (5.710) | | | | (-3.107) | [0.000] | |
| (5) | 1.126 | -0.066 | -2.818 | 8.808 | 2.838 | 21.913 | 13.933 | | | | 43.186 | 0.633 |
| | (10.086) | (-1.805) | (-0.481) | (0.144) | (1.573) | (3.937) | (2.450) | | | | [0.056] | |
| (6) | 0.835 | -0.008 | -5.807 | 15.057 | 3.514 | 17.502 | | -7.773 | | | 48.943 | 0.699 |
| | (9.206) | (-1.213) | (-1.280) | (0.154) | (1.569) | (3.064) | | (-1.850) | | | [0.141] | |
| (7) | 1.093 | -0.011 | 1.002 | -67.713 | 2.490 | 18.722 | | | 0.655 | | 63.720 | 0.683 |
| | (8.573) | (-0.355) | (0.154) | (-1.176) | (0.824) | (3.239) | | | (0.424) | | [0.000] | |
| (8) | 1.146 | -0.029 | 1.671 | -9.296 | 2.904 | 16.517 | | | | -5.846 | 49.337 | 0.647 |
| | (13.055) | (-0.999) | (0.286) | (-0.162) | (1.461) | (3.711) | | | | (-1.386) | [0.015] | |

Table 15: Linear GMM Tests with Tail Risk

This table reports results of GMM tests of pricing kernels using the combination of 6 ATM put portfolios and 6 ATM call portfolios as test assets. Each row represents a model and columns represent factors included in the model. The point estimates are reported along with t-statistics in parentheses that are computed using Newey-West adjusted standard errors with 6-month lags. The final two columns give the J-statistic with corresponding asymptotic p-value in [brackets] and the Hansen-Jagannathan measure of distance from the space of valid stochastic discount factors.

**ATM Portfolios**

| | intercept | MKT | SMB | HML | MOM | VOL | DS | SKEW | JUMP | VOL JUMP | Jstat | HJ dist |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| (1) | 1.136 | -0.047 | | | | 19.843 | 10.239 | | | | 15.288 | 0.291 |
| | (7.690) | (-1.011) | | | | (2.245) | (1.081) | | | | [0.083] | |
| (2) | 0.983 | -0.003 | | | | 17.005 | | -0.318 | | | 19.472 | 0.290 |
| | (10.972) | (-0.140) | | | | (2.688) | | (-0.144) | | | [0.021] | |
| (3) | 0.994 | -0.010 | | | | 16.040 | | | -5.008 | | 17.597 | 0.325 |
| | (18.490) | (-0.689) | | | | (2.409) | | | (-2.132) | | [0.057] | |
| (4) | 1.129 | -0.023 | | | | 14.137 | | | | -3.710 | 16.065 | 0.315 |
| | (10.510) | (-1.041) | | | | (3.062) | | | | (-0.986) | [0.066] | |
| (5) | 1.174 | -0.084 | 5.997 | 72.356 | -1.979 | 24.059 | 15.124 | | | | 8.504 | 0.293 |
| | (6.358) | (-1.492) | (0.649) | (0.612) | (-0.302) | (2.108) | (1.260) | | | | [0.203] | |
| (6) | 0.975 | -0.005 | -0.120 | -2.903 | 3.374 | 24.108 | | -4.972 | | | 9.871 | 0.298 |
| | (10.597) | (-0.233) | (-0.010) | (-0.033) | (0.515) | (2.550) | | (-1.870) | | | [0.218] | |
| (7) | 0.933 | -0.023 | 3.650 | 13.343 | -9.402 | 11.971 | | | -0.888 | | 16.309 | 0.319 |
| | (10.392) | (-0.877) | (0.395) | (0.152) | (-2.419) | (1.935) | | | (-0.592) | | [0.012] | |
| (8) | 0.990 | -0.042 | 4.680 | 63.669 | -6.074 | 15.034 | | | | -0.268 | 11.417 | 0.336 |
| | (7.529) | (-1.435) | (0.527) | (0.701) | (-1.402) | (2.096) | | | | (-0.063) | [0.076] | |

Table 16: Linear GMM Tests with Tail Risk

This table reports results of GMM tests of pricing kernels using 12 stock portfolios as test assets. Each row represents a model and columns represent factors included in the model. The point estimates are reported along with t-statistics in parentheses that are computed using Newey-West adjusted standard errors with 6-month lags. The final two columns give the J-statistic with corresponding asymptotic p-value in [brackets] and the Hansen-Jagannathan measure of distance from the space of valid stochastic discount factors.

**12 Stock Portfolios**

| | intercept | MKT | SMB | HML | MOM | VOL | DS | SKEW | JUMP | VOL JUMP | Jstat | HJ dist |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| (1) | 1.025 | -0.049 | | | | 13.116 | 4.340 | | | | 19.652 | 0.259 |
| | (11.794) | (-1.611) | | | | (2.077) | (0.675) | | | | [0.186] | |
| (2) | 0.958 | -0.036 | | | | 13.527 | | -2.353 | | | 20.648 | 0.383 |
| | (19.566) | (-2.044) | | | | (1.831) | | (-1.094) | | | [0.148] | |
| (3) | 0.991 | -0.039 | | | | 15.546 | | | 0.445 | | 20.668 | 0.188 |
| | (21.655) | (-2.209) | | | | (2.159) | | | (0.448) | | [0.148] | |
| (4) | 1.001 | -0.042 | | | | 12.859 | | | | -1.497 | 19.939 | 0.276 |
| | (9.096) | (-1.954) | | | | (1.874) | | | | (-0.386) | [0.174] | |
| (5) | 1.022 | -0.056 | 0.716 | 5.890 | 0.515 | 13.858 | 4.026 | | | | 18.971 | 0.269 |
| | (6.188) | (-0.860) | (0.174) | (0.127) | (0.163) | (2.438) | (0.379) | | | | [0.089] | |
| (6) | 1.009 | -0.054 | -4.181 | -61.357 | -4.002 | 9.870 | | -5.858 | | | 18.032 | 0.229 |
| | (12.897) | (-1.762) | (-0.716) | (-0.732) | (-0.857) | (1.213) | | (-1.282) | | | [0.115] | |
| (7) | 0.981 | -0.038 | -0.086 | -8.364 | 0.245 | 11.648 | | | 0.369 | | 19.014 | 0.209 |
| | (14.028) | (-1.264) | (-0.021) | (-0.212) | (0.103) | (1.663) | | | (0.293) | | [0.088] | |
| (8) | 0.969 | -0.033 | -0.773 | -6.840 | -1.276 | 10.114 | | | | -0.749 | 18.379 | 0.281 |
| | (2.504) | (-0.626) | (-0.074) | (-0.058) | (-0.322 | (1.595) | | | | (-0.065) | [0.105] | |

Table 17: Linear GMM Tests with Tail Risk

This table reports results of GMM tests of pricing kernels using the 12 ATM option portfolios combined with the 12 stock portfolios as test assets. Each row represents a model and columns represent factors included in the model. The point estimates are reported along with t-statistics in parentheses that are computed using Newey-West adjusted standard errors with 6-month lags. The final two columns give the J-statistic with corresponding asymptotic p-value in [brackets] and the Hansen-Jagannathan distance measure.

**12 ATM Options Portfolios and 12 Stock Portfolios**

| | intercept | MKT | SMB | HML | MOM | VOL | DS | SKEW | JUMP | VOL JUMP | Jstat | HJ dist |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| (1) | 1.116 | -0.070 | | | | 15.024 | 14.377 | | | | 67.400 | 0.488 |
| | (14.535) | (-2.592) | | | | (3.819) | (2.772) | | | | [0.000] | |
| (2) | 0.914 | -0.018 | | | | 16.228 | | -6.837 | | | 60.283 | 0.503 |
| | (26.498) | (-1.401) | | | | (4.171) | | (-2.990) | | | [0.000] | |
| (3) | 1.055 | -0.046 | | | | 10.216 | | | 0.417 | | 91.981 | 0.512 |
| | (14.670) | (-2.069) | | | | (2.575) | | | (0.262) | | [0.000] | |
| (4) | 0.986 | -0.036 | | | | 8.664 | | | | -1.638 | 71.371 | 0.499 |
| | (11.851) | (-2.298) | | | | (2.812) | | | | (-0.532) | [0.000] | |
| (5) | 1.098 | -0.077 | 5.106 | -154.740 | 0.127 | 12.847 | 13.863 | | | | 49.322 | 0.565 |
| | (8.849) | (-2.008) | (0.867) | (-2.738) | (0.044) | (2.822) | (1.928) | | | | [0.000] | |
| (6) | 0.881 | -0.028 | 0.848 | -127.291 | -0.673 | 15.976 | | -7.858 | | | 45.738 | 0.537 |
| | (16.309) | (-1.959) | (0.240) | (-2.294) | (-0.247) | (4.210) | | (-3.808) | | | [0.000] | |
| (7) | 1.121 | -0.067 | 1.725 | -144.936 | -3.695 | 7.742 | | | 0.404 | | 71.455 | 0.554 |
| | (13.555) | (-2.581) | (0.347) | (-2.613) | (-1.338) | (1.836) | | | (0.214) | | [0.000] | |
| (8) | 1.008 | -0.052 | -2.301 | -96.950 | -3.626 | 8.493 | | | | -0.476 | 61.052 | 0.532 |
| | (10.099) | (-2.677) | (-0.937) | (-2.348) | (-1.860) | (2.403) | | | | (-0.111) | [0.000] | |

Table 18: GMM Likelihood Ratio-type tests

This table reports results of GMM-based likelihood ratio-type tests of restricting individual factors to be the same in both the stock SDF and the SDF estimated from option portfolios. Each pair of numbers represents a test where the model is estimated first under the restriction that the prices of risk for all risk factors in each model are the same for stocks and call options. This corresponds to the results in Table 9. This restriction is relaxed for volatility factor to estimate the unrestricted model. The values in the table are the test statistic and corresponding p-values in brackets. The null hypothesis is that the price of risk for the volatility factor are the same in options and stocks. The alternative is that the price of risk differs between the two markets.

**Likelihood Ratio tests**

| **One Factor Model** | | **Two Factor Model** | | **Five Factor Model** | |
|---|---|---|---|---|---|
| Test Statistic | p-value | Test Statistic | p-value | Test Statistic | p-value |
| 3.557 | 0.059 | 3.002 | 0.083 | 4.698 | 0.030 |


A) Volatility (innovations)

B) Orthogonal Volatility (innovations)

C) Down Side

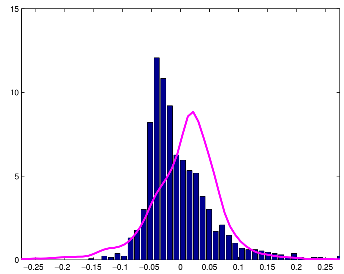
D) Model-Free Skew (innovations)

E) Jump

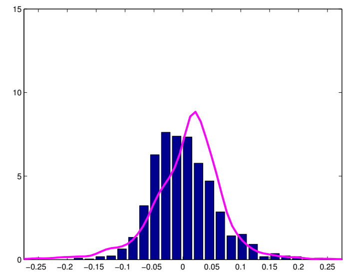
F) Volatility Jump

Figure 1: Factors

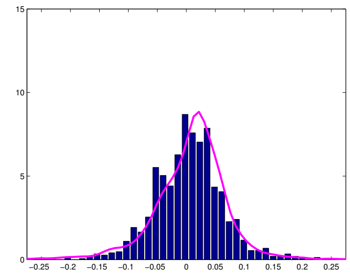
Panel A plots innovations in the VIX. Panel B plots the time series of residuals from regressing VIX innovations on market excess returns (MKT). This is the orthogonalized volatility factor used in tests throughout the paper. The time series plotted in each Panels C, D, E and F represent tail risk factors.



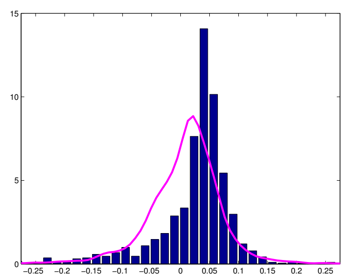
A) OTM Calls



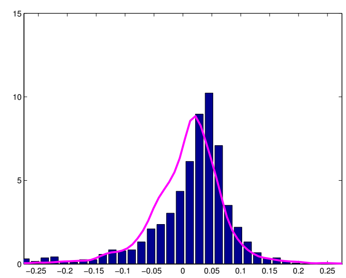
B) ATM Calls



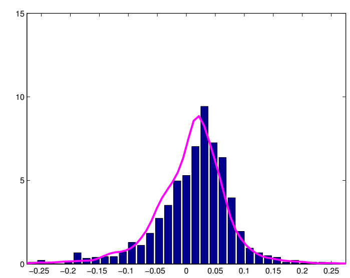
C) ITM Calls



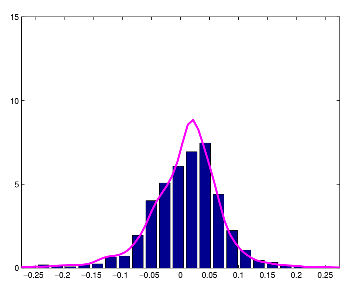
D) OTM Puts



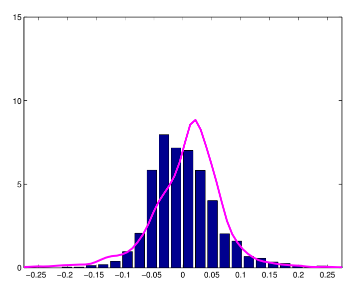
E) ATM Puts



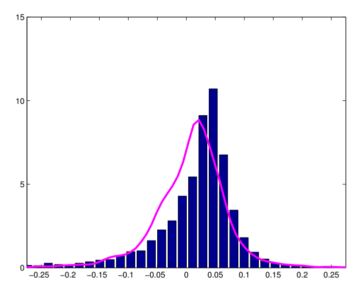
F) ITM Puts



G) ATM Calls and Puts



H) All calls



I) All Puts

Figure 2: Empirical densities of moneyness, put/call portfolios

Panels A-F plot the empirical densities of OTM Calls, ATM Calls, ITM Calls, OTM Puts, ATM Puts and ITM Puts respectively. The horizontal axis measures monthly returns and the vertical axis measures density of the distribution. Each panel has a kernel density estimate of the realized returns for the 12 stock portfolios overlaying the empirical density for comparision. Each empirical density in panels A-F is composed of 1,200 observed returns; 200 monthly holding period returns from each of the 6 implied volatility premium portfolios within a given moneyness-put/call portfolio. Panel G plots the combined ATM calls and ATM puts. Panels H and I plot the empirical densities of all call portfolios and all put portfolios respectively, across all three moneyness bins.